CFD Based Aeroelastic Stability Predictions Under the Influence of Structural Variability

S. Marques
CONTENTS

• Motivation

• Schur Complement Method

• Perturbation Analysis

• Interval Analysis

• Results

• Conclusions
MOTIVATION

• Uncertainty can have many sources

• Transonic Aerodynamics
  - Structural Variability
MOTIVATION

• Methods to account for variability
  
  • Probabilistic Methods: MC; Perturbation Methods

  • Non-Probabilistic: Interval Analysis
MOTIVATION

• Methods to account for variability
  • Probabilistic Methods: MC; Perturbation Methods
  • Non- Probabilistic: Interval Analysis
• Transonic Aerodynamics requires a departure from linear methods
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• Transonic Aerodynamics requires a departure from linear methods
  – CFD
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• Methods to account for variability
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• Transonic Aerodynamics requires a departure from linear methods
  – CFD

• Challenge: How to integrate variability with CFD based computational aeroelasticity, in a feasible and efficient way?
SCHUR METHOD

• The coupled CFD-CSD system can be described as:

\[
\frac{d\mathbf{w}}{dt} = \mathbf{R}(\mathbf{w}, \mu)
\]
SCHUR METHOD

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\[
\frac{dw}{dt} = R(w, \mu)
\]

\[
w = [w_f, w_s]^T;
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\[
R = [R_f, R_s]^T
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\[
\mu - \text{Bifurcation Parameter}
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Computationally very intensive
Impractical for flight envelope search
The coupled CFD-CSD system can be described as:

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\mathbf{R} = [\mathbf{R}_f, \mathbf{R}_s]^T
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Computationally very intensive
Impractical for flight envelope search

However…

Pitch and Plunge Aerofoil Case
SCHUR METHOD

• The eigenvalue problem can be written as:

\[
\begin{bmatrix}
A_{ff} & A_{fs} \\
A_{sf} & A_{ss}
\end{bmatrix}
\begin{bmatrix}
p_f \\
p_s
\end{bmatrix} = \lambda
\begin{bmatrix}
p_f \\
p_s
\end{bmatrix}
\]
SCHUR METHOD

• The eigenvalue problem can be written as:

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\frac{\partial R_f}{\partial W_s}
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= \lambda
\begin{bmatrix}
p_f \\
p_s
\end{bmatrix}
\]

• Shifted Inverse Power Method
  — System becomes ill-conditioned
  — Solving in Parallel Difficult

\[
z_k = \left[ \begin{array}{cc}
A_{ff} - \lambda_0 I & A_{fs} \\
A_{sf} & A_{ss} - \lambda_0 I
\end{array} \right]^{-1} x_{k-1}
\]

Badcock et al, AIAA J, 45(6), 1370-1381, 2007
SCHUR METHOD

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• Schur Complement formulation:

\[S(\lambda)p_s = \lambda p_s\]

\[S(\lambda) = A_{ss} - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs}\]

\(\lambda\) is not an eigenvalue of \(A_{ff}\)

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New formulation for Non-linear Eigenvalue Problem

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SCHUR METHOD

• The new formulation is solved by Newton’s Method

\[
\frac{\partial F}{\partial u} \Delta u = -F
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SCHUR METHOD

• The new formulation is solved by Newton’s Method

\[ \frac{\partial F}{\partial u} \Delta u = -F \]

\[
\begin{bmatrix}
S(\lambda) - \lambda I & \frac{\partial S(\lambda)}{\partial \lambda} p_s - p_s \\
q & 0
\end{bmatrix}
\]

\[ S(\lambda)p_s - \lambda p_s = F \]

\[ S(\lambda) = A_{ss} - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs} \]

\[ u = [p_s \quad \lambda]^T \]
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Full Evaluation, Expensive
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S(\lambda) = A_{ss} - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs}
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But

\[
(A_{ff} - \lambda I)^{-1} \approx A_{ff}^{-1} + \lambda A_{ff}^{-1} A_{ff}^{-1} + \lambda^2 A_{ff}^{-1} A_{ff}^{-1} A_{ff}^{-1} + \ldots
\]

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\]

Pre-Compute

\[
A_{sf} A_{ff}^{-1} A_{fs} \quad \text{and} \quad A_{sf} A_{ff}^{-2} A_{fs}
\]

PERTURBATION METHOD

Generate a new set of Parameters

MatLab script
PERTURBATION METHOD

Generate a new set of Parameters

MSC Nastran input modes

MatLab script
PERTURBATION METHOD

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MSC Nastran input modes

Np Schur Calculations

MatLab script
PERTURBATION METHOD

Generate a new set of Parameters

MSC Nastran input modes

Np Schur Calculations

Perturbation Analysis

MatLab script
INTERVAL ANALYSIS

\[
\left[ \alpha(\theta)_i, \bar{\alpha}(\theta)_i \right] = [\min(\lambda_i), \max(\lambda_i)]
\]

\[
S(\lambda)p_s - \lambda p_s = 0, \forall i
\]

\[
\theta \leq \theta \leq \bar{\theta}
\]
INTERVAL ANALYSIS

\[
\begin{align*}
\left[\underline{\lambda}(\theta)_i, \overline{\lambda}(\theta)_i\right] &= [\min(\lambda_i), \max(\lambda_i)] \\
S(\lambda)p_s - \lambda p_s &= 0, \forall i \\
\theta &\leq \underline{\theta} \leq \overline{\theta}
\end{align*}
\]

• Optimisation Problem
INTERVAL ANALYSIS

\[ \begin{align*}
\lambda(\theta)_i, \bar{\lambda}(\theta)_i &= [\min(\lambda_i), \max(\lambda_i)] \\
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\end{align*} \]

- Optimisation Problem

- Objective Function Calculation – Schur Method
INTERVAL ANALYSIS - OPTIMISER

- Optimiser – SQP – Lagrangian Hessian based algorithm

\[ \left\| \nabla_{\theta} L(\theta, \varphi) \right\| = \left\| \nabla \lambda(\theta) + \sum_{j=1}^{N_p} \varphi_j \nabla g_j(\theta) \right\| \]
INTERVAL ANALYSIS - OPTIMISER

- Optimiser – SQP – Lagrangian Hessian based algorithm

\[
\left\| \nabla_{\theta} L(\theta, \varphi) \right\| = \left\| \nabla \lambda(\theta) - \sum_{j=1}^{N_p} \varphi_j \nabla g_j(\theta) \right\|
\]

Requires one Schur evaluation
INTERVAL ANALYSIS - OPTIMISER

- Optimiser – SQP – Lagrangian Hessian based algorithm

\[ \nabla_{\theta} L(\theta, \varphi) = \nabla \lambda(\theta) + \sum_{j=1}^{N_p} \varphi_j \nabla g_j(\theta) \]

Requires \( N_p \) Schur evaluations
INTERVAL ANALYSIS - OPTIMISER

Estimate / Update
Hessian
INTERVAL ANALYSIS - OPTIMISER

1. Calculate new modes from MSC Nastran

Estimate / Update Hessian
INTERVAL ANALYSIS - OPTIMISER

1. Calculate new modes from MSC Nastran
2. Calculate obj. function gradient – eigenvalue – 1 Schur calculation

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INTERVAL ANALYSIS - OPTIMISER

1. Calculate new modes from MSC Nastran

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Estimate / Update Hessian

Find search direction / Calculate contraints gradient
INTERVAL ANALYSIS - OPTIMISER

1. Calculate new modes from MSC Nastran
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1. Find search direction / Calculate contraints gradient
2. Estimate / Update Hessian

1. Calculate new modes from MSC Nastran
INTERVAL ANALYSIS - OPTIMISER

1. Calculate new modes from MSC Nastran
2. Calculate obj. function gradient – eigenvalue – 1 Schur calculation

1. Calculate new modes from MSC Nastran
2. Np Schur calculations
GOLAND WING – CLEAN CASE

- 7 Structural Parameters
  - Variation within ±5%
  - 1000 MC Samples Generated
GOLAND WING – MODE TRACKING

![Graphs showing mode tracking for different modes and altitudes.]
GOLAND WING – MODE TRACKING

• 40k pts grid
• Tracking 4 modes
• 1 Workstation < 12 minutes;
  • Steady State – 1 min
  • $A_{sf} A_{ff}^{-1} A_{fs}$ and $A_{sf} A_{ff}^{-2} A_{fs}$ - 10 min
  • Envelope Sweep < 1 min; 5 Full Evaluations- 25 min
GOLAND WING – MODE TRACKING

- MC Analysis – 1000 samples
- As instability approaches, variability increases
GOLAND WING – MODE TRACKING

• Uncertainty – MC, Perturbation Method, Interval Analysis
GOLAND WING – MODE TRACKING

• Uncertainty – MC, Perturbation Method, Interval Analysis
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GOLAND WING – METHODS

• Structural variability increases uncertainty as mode becomes unstable
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  • Perturbation Method provides a good estimate, at low cost, of such effects. However this estimate is not conservative
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  • Perturbation Method provides a good estimate, at low cost, of such effects. However this estimate is not conservative

  • Interval Analysis, gives conservative estimates of the effects of structural variability, but at a higher cost than perturbation methods
GOLAND WING – METHODS

• Structural variability increases uncertainty as mode becomes unstable

• Perturbation Method provides a good estimate, at low cost, of such effects. However this estimate is not conservative

• Interval Analysis, gives conservative estimates of the effects of structural variability, but at a higher cost than perturbation methods

<table>
<thead>
<tr>
<th>Method</th>
<th>N. of Eigenvalue Calculations – 7Param</th>
<th>Wall Clock Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC</td>
<td>1000</td>
<td>50h</td>
</tr>
<tr>
<td>Perturbation</td>
<td>7</td>
<td>21min</td>
</tr>
<tr>
<td>Interval Analysis</td>
<td>60 - 200</td>
<td>2 – 8h</td>
</tr>
<tr>
<td>Single Eigv eval.</td>
<td>1</td>
<td>3min</td>
</tr>
</tbody>
</table>
GOLAND WING – STORE CASE

Mach 0.70
GOLAND WING – STORE CASE

Mach 0.70

Mach 0.80
ECERTA Advisory Board Meeting - 2009

GOLAND WING – STORE CASE

Mach 0.85

Mach 0.90
GOLAND WING – STORE CASE

Mach 0.97
GOLAND WING – STORE CASE

Mach 0.90 - MC Samples
GOLAND WING – STORE CASE

Mach 0.90 Instability detail
Instability Boundary
CONCLUSION

• A very fast method to calculate flutter boundary has been developed
  – The method is easily parallelised
  – It allows for mode tracking at all conditions
  – Series approximation efficient and accurate
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  — The method is easily parallelised
  — It allows for mode tracking at all conditions
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• CFD has been integrated providing an uncertainty quantification method

• Structural variability can have a determinant effect on flutter margins
FUTURE WORK

• Expand Generic Fighter Flutter Envelope and Structural UQ study
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59% span

85% span
FUTURE WORK

3.920 Hz

9.191 Hz

9.964 Hz

22.452 Hz

22.608 Hz

24.020 Hz

26.772 Hz

31.292 Hz

40.04 Hz

41.695 Hz
FUTURE WORK

Mach 0.85; AoA 0°

- 32 Processors
- Steady State – 15 min
- $A_{sf}A^{-1}_{ff}A_{fs}$ and $A_{sf}A^{-2}_{ff}A_{fs}$ for 8 Modes - 10 Hours
FUTURE WORK

• Expand Generic Fighter Flutter Envelope and Structural UQ study

• Introduce Effects of Atmospheric Uncertainty on Flutter
  – Flight tests at the same conditions have shown significant variability in results, which may be attributed to atmospheric unknown conditions

Thank you for your attention.
Any Questions?