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A member of the Russell Group

Linear Reduced Order Model for Gust Loads Prediction using the DLR-TAU Code

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Motivation

Unified Framework for Aeroelastic Analyses using CFD Tools

Stability Problems

Fast Flutter Method

- Demonstration on full scale Airbus production aircraft done earlier this year

Response Problems

NROM for Gust Analysis

- Straightforward extension of Fast Flutter Method
- Soon demonstration

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Fast Flutter Method

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Aeroelastic Eigenvalue Problem

$$\left(\begin{bmatrix}0 & I\\ -K_\eta & -C_\eta\end{bmatrix} + Q(\omega_j)\right)\mathbf{p}_s^j = \lambda_j \mathbf{p}_s^j \quad \text{for } j=1,\ldots,n$$

n: number of structural modes

$$Q(\omega) = -A_{sf}(A_{ff} - i\omega I)^{-1}(A_{f\eta} + i\omega A_{f\dot\eta})$$

pre-computed samples of *Q*

$$(A_{ff} - i\omega I)Y = A_{f\eta} + i\omega A_{f\dot\eta}$$

n linear solves per sample

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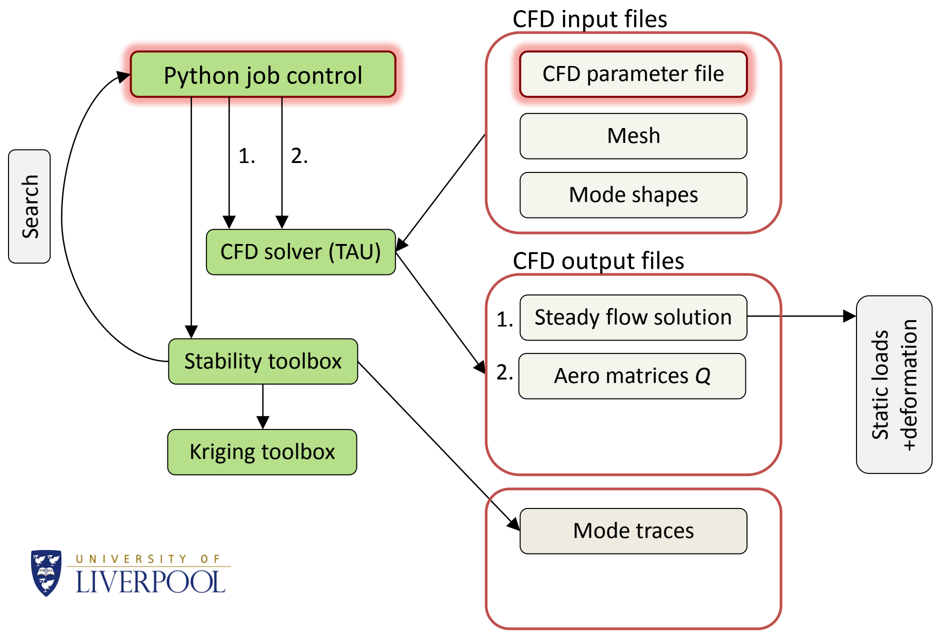
Airbus Demonstration – Basic Flutter Process



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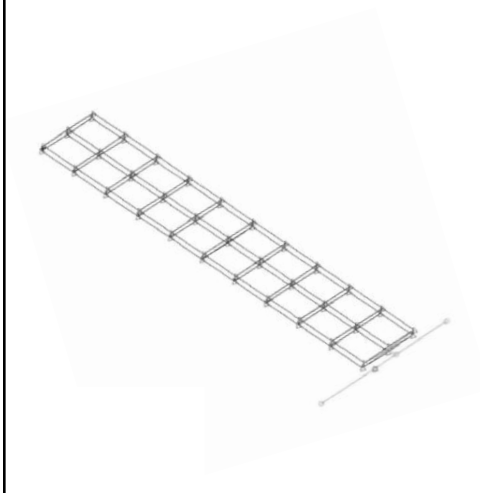
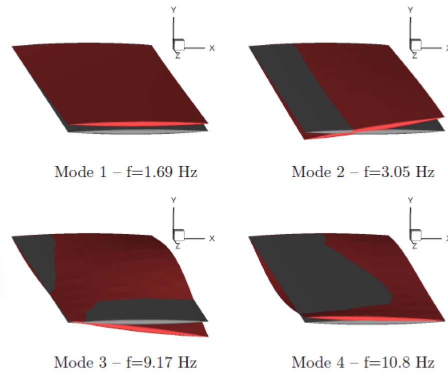
Goland Wing/Store Case

- Euler simulation
- mesh with 400,000 points
- four elastic modes
- no structural damping





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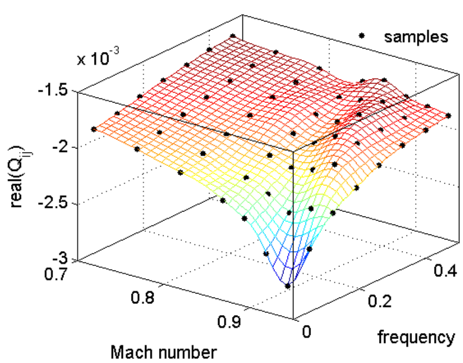
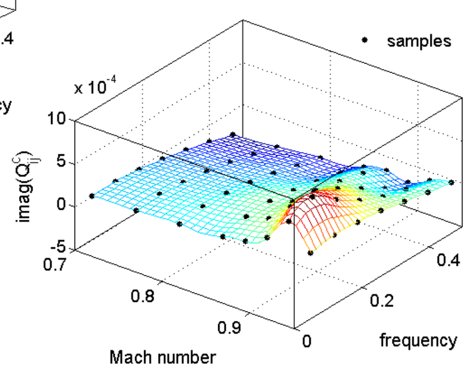
Goland Wing/Store Case



Element of Aero matrix *Q*



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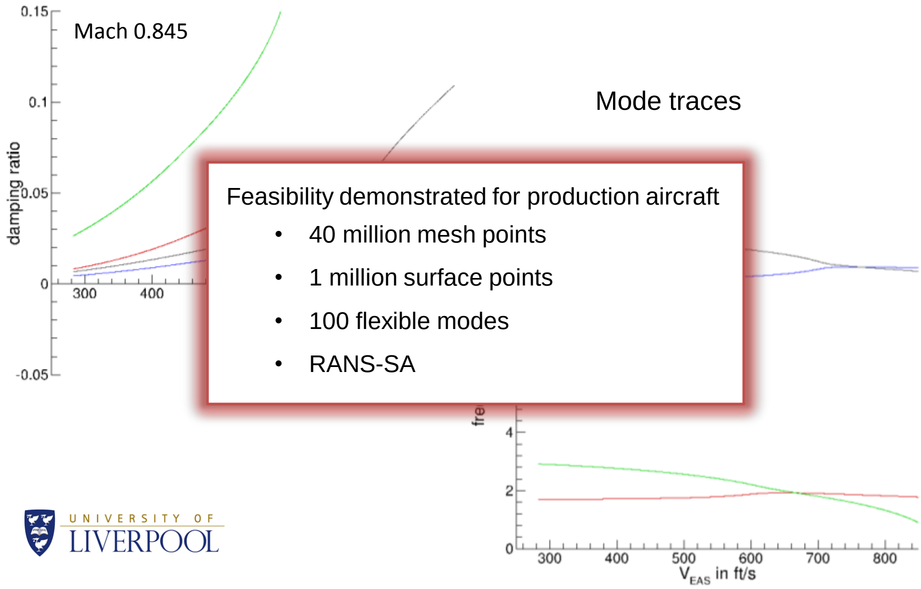

Feasibility demonstrated for production aircraft

- 40 million mesh points
- 1 million surface points
- 100 flexible modes
- RANS-SA

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ROM for Gust Analysis

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ROM for Gust Analysis

Da Ronch et al., AIAA Paper 2012-4404

$$\dot{\boldsymbol{w}} = \boldsymbol{f}(\boldsymbol{w}, \boldsymbol{u}_g)$$

millions of DOFs

$$\boldsymbol{w} = \boldsymbol{w}_0 + \Delta\boldsymbol{w}$$

$$\Delta\dot{\boldsymbol{w}} = \boldsymbol{f}(\boldsymbol{w}_0) + A\Delta\boldsymbol{w} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}_g}\Delta\boldsymbol{u}_g + H.O.T.$$

$$\Delta\boldsymbol{w} = \Phi\boldsymbol{z} + \bar{\Phi}\bar{\boldsymbol{z}}$$

$$\bar{\Psi}^T\Phi = I$$

$$\bar{\Psi}^T\bar{\Phi} = 0$$

$$\dot{\boldsymbol{z}} = \Lambda\boldsymbol{z} + \bar{\Psi}^T\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}_g}\Delta\boldsymbol{u}_g$$

n DOFs

Λ: diagonal eigenvalue matrix

Φ: right modal matrix

Ψ: left modal matrix

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Connection with Fast Flutter Method

- ROM requires eigenvalues and right and left eigenvectors

$$\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n),\ \Phi = [\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_n] \text{ and } \Psi = [\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_n]$$

$$\text{where } \boldsymbol{p} = [\boldsymbol{p}_f^T, \boldsymbol{p}_s^T]^T \text{ and } \boldsymbol{q} = [\boldsymbol{q}_f^T, \boldsymbol{q}_s^T]^T$$

- Basic Fast Flutter Method provides
  - right structural eigensolution

    λ and $\boldsymbol{p}_s$

  - left structural eigensolution; comes at "no" extra cost

    λ and $\boldsymbol{q}_s$

$$Q = -A_{sf}(A_{ff} - i\omega I)^{-1}(\cancel{A_{f\eta} + i\omega A_{f\dot\eta}})$$

A_{fs}

$$\boldsymbol{p}_s = \begin{bmatrix} \boldsymbol{p}_\eta \\ \lambda\boldsymbol{p}_\eta \end{bmatrix}, \text{ but not } \boldsymbol{q}_s \neq \begin{bmatrix} \boldsymbol{q}_\eta \\ \lambda\boldsymbol{q}_\eta \end{bmatrix}$$

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Connection with Fast Flutter Method

- Extend the basic method to calculate the fluid eigenvectors

$$\boldsymbol{p}_f^j = -(A_{ff} - \lambda_j I)^{-1} A_{fs} \boldsymbol{p}_s^j \quad \text{for} \quad j = 1, \ldots, n$$

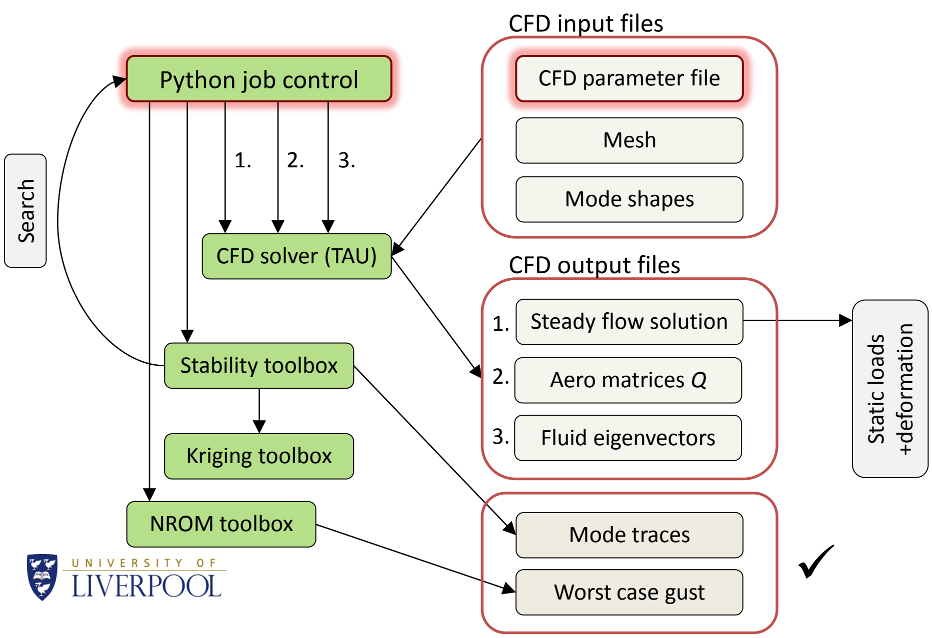
$$\boldsymbol{q}_f^j = -(A_{ff}^T - \lambda_j I)^{-1} A_{sf}^T \boldsymbol{q}_s^j \quad \text{for} \quad j = 1, \ldots, n$$

Additional cost of 2n linear solves

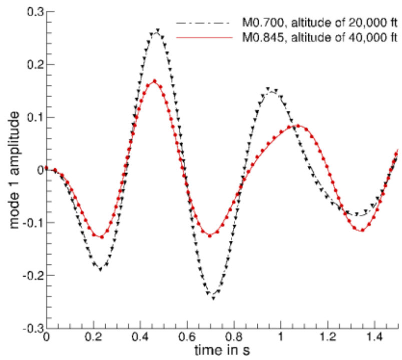
- ROM also needs gust influence vector; done with a finite difference

$$\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}_g} = \frac{\boldsymbol{f}(\dot{\boldsymbol{x}} + \varepsilon \boldsymbol{u}_g) - \boldsymbol{f}(\dot{\boldsymbol{x}} - \varepsilon \boldsymbol{u}_g)}{2\varepsilon}$$

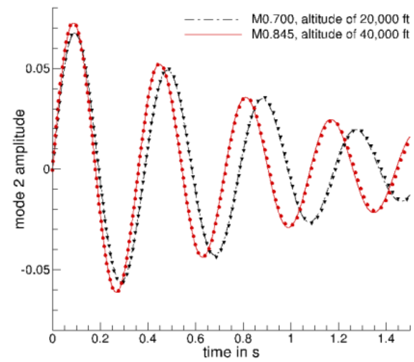
Airbus Demonstration – Extended Process



Goland Wing/Store Case



free response



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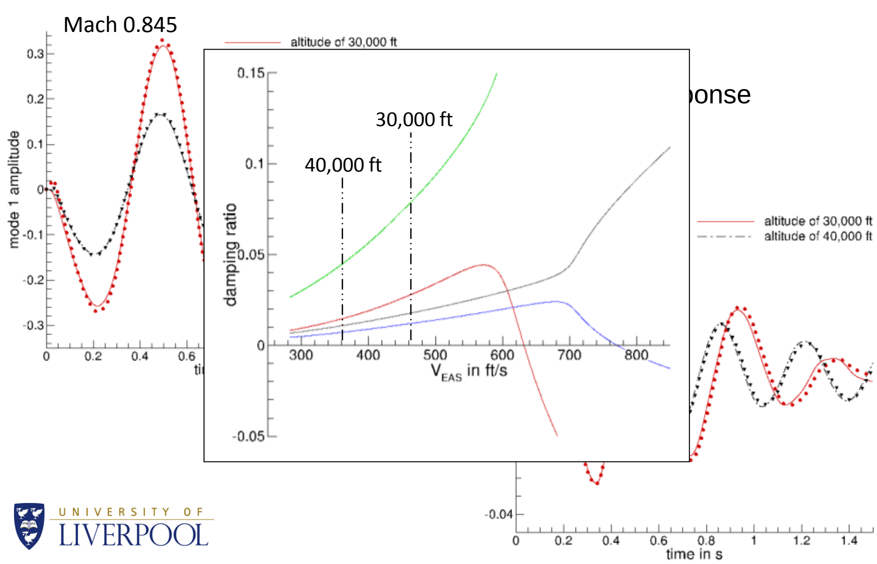
Goland Wing/Store Case

- 1-minus-cos
- h_g=6.25
- w_g=0.01



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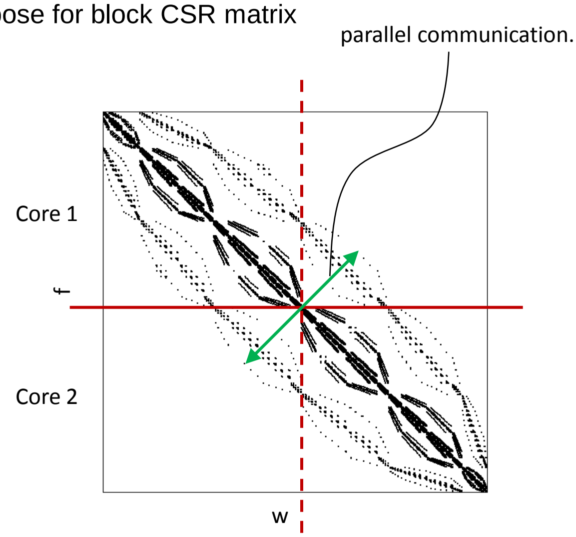
New TAU Developments

- Parallel transpose for block CSR matrix



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New TAU Developments

- Parallel transpose for block CSR matrix
- TAU LFD solver: it is all about solving linear systems

  MG – GMRes → block ILU$^{\mathfrak{R}}$ – GCR → block ILU$^{\mathfrak{C}}$ – GMRes

  - Outperforms MG-GMRes significantly
  - Still works, when MG-GMRes fails
  - Expected for next TAU release

- Matrix A_{sf}
  - was already formed element-wise when integrating surface LFD solution

$$Q = -A_{sf}(A_{ff} - i\omega I)^{-1} A_{fs}$$

  - now written to disk to form RHS for left eigenvector calculations

$$(A_{ff}^T - \lambda_j I)\boldsymbol{q}_f^j = -A_{sf}^T \boldsymbol{q}_s^j \quad \text{for} \quad j = 1, \ldots, n$$

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Suggested Future TAU Developments

- Main cost is in solving linear systems
  - Better initialisation of solution $\boldsymbol{x}_0$ to solve: $A\boldsymbol{x} = \boldsymbol{b}$
  - TAU LFD solver:
    MG – GMRes → block $ILU^{\mathfrak{R}}$ – GCR → block $ILU^{\mathfrak{C}}$ – GMRes
    - Currently: form A_{ff} and block ILU of A_{ff}, then solve for <u>one</u> RHS $\boldsymbol{b}$
    - Better: solve for <u>multiple</u> RHSs (i.e. for all modes) in one go
  - Apply interpolation for right fluid eigenvectors

    Cost savings up to 50% possible !?!

    $$Y = -(A_{ff} - i\omega I)^{-1} A_{fs} \quad \rightarrow \quad \boldsymbol{p}_f^j = -(A_{ff} - \lambda_j I)^{-1} A_{fs} \boldsymbol{p}_s^j$$
- Gust influence matrix for random and localised gust excitation

$$\overline{\Psi}^T \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}_g} \quad \rightarrow \quad \overline{\Psi}^T \frac{\partial \boldsymbol{f}}{\partial \dot{\boldsymbol{x}}} \frac{\partial \dot{\boldsymbol{x}}}{\partial \boldsymbol{u}_g}$$

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Outlook

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Outlook

- ROM for gust analysis implemented using DLR-TAU solver
- Extend residual expansion to account for altitude variation

$$\Delta\dot{\boldsymbol{w}} = \boldsymbol{f}(\boldsymbol{w}_0) + A\Delta\boldsymbol{w} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}_g}\Delta\boldsymbol{u}_g + \frac{\partial \boldsymbol{f}}{\partial H}\Delta H + H.O.T.$$

- Demonstration for full aircraft model – XRF case

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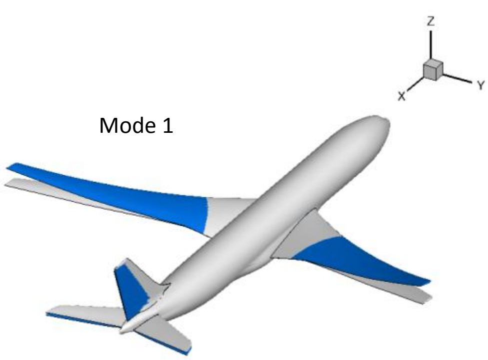
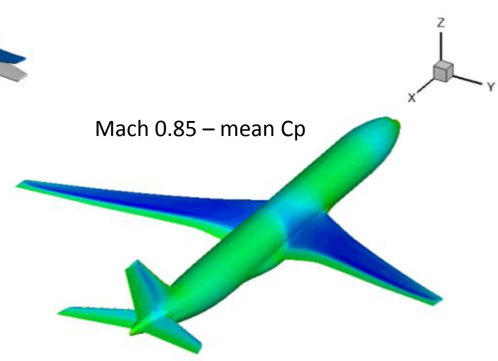
XRF Case

- Euler simulation
- mesh with 800,000 points
- 15 elastic modes
- no structural damping





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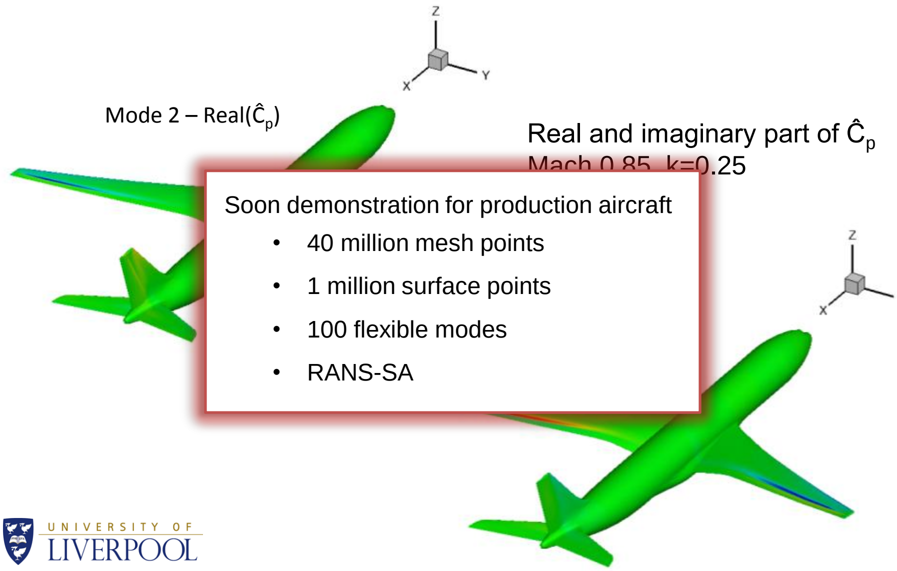
XRF Case

Mode 2 – Real($\hat{C}_p$)

Real and imaginary part of $\hat{C}_p$
Mach 0.85, k=0.25



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