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Application of Harmonic Balance Method for Non-linear Gust Responses

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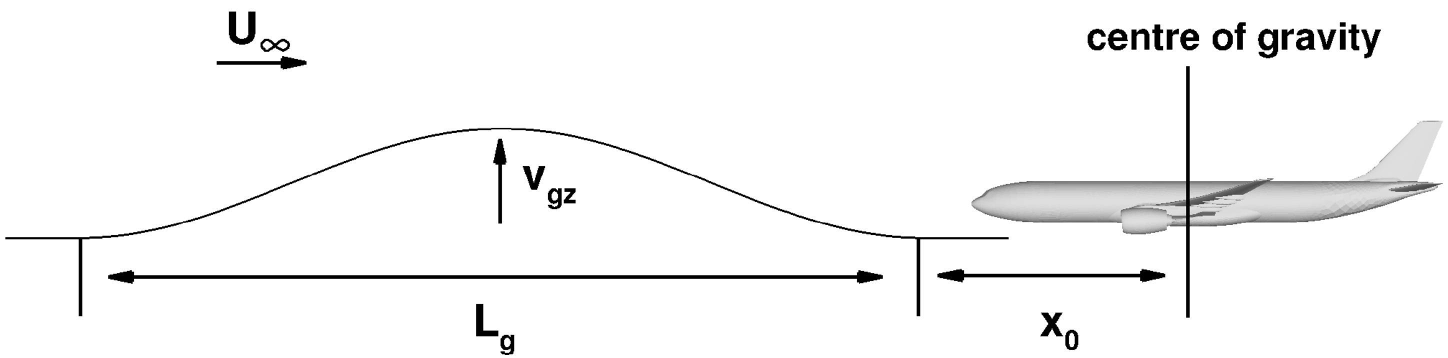
Motivation

- Gust analysis one challenge in certification
- Covering a large parameter space
- Linear potential methods (DLM) fail in transonic regime
- Non-linear RANS equations coupled to structure and flight dynamics computationally too expensive
- Linearised RANS methods retain RANS accuracy at significantly reduced cost



Linearised Frequency-Domain: A Short Introduction

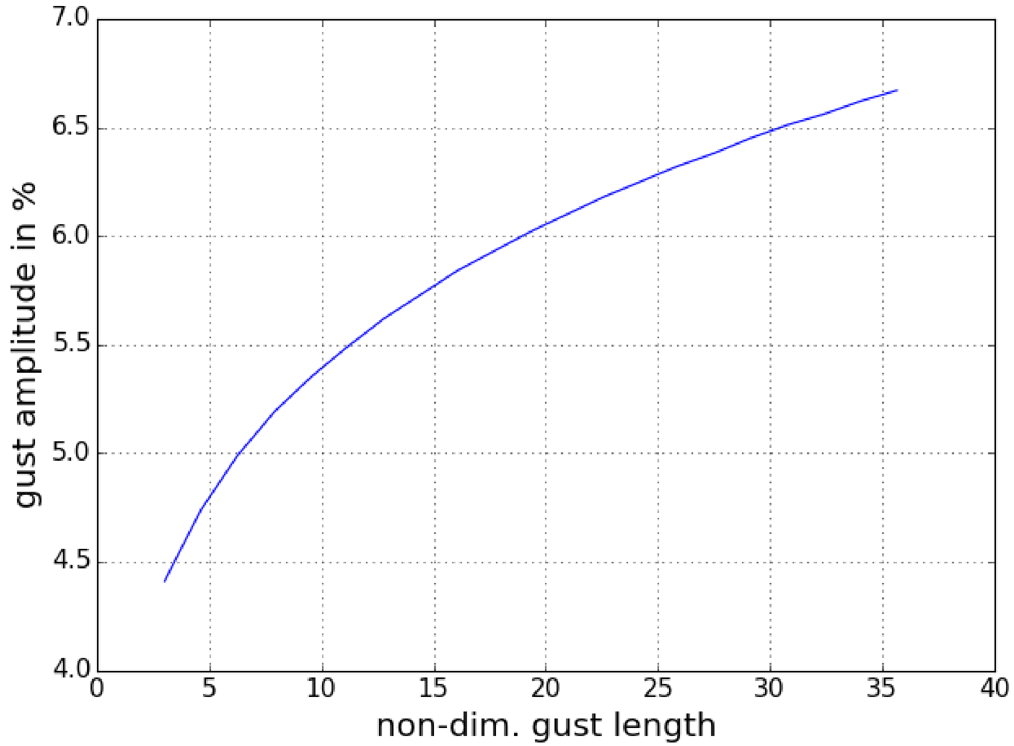
- Starting with spatially discretised RANS equations
- Separate variables in steady mean state and small time-dependent perturbation
- Linearise non-linear residual function around steady flow-field
- Transform equation into frequency domain
- Obtain a large, but sparse system of linear equations

$$\left(\frac{\partial R}{\partial W} - j\omega I\right)\widehat{W} = -\frac{\partial R}{\partial v_g}\hat{v}_g$$

Frequency-Domain Non-linear Gust Response Computation

Motivation

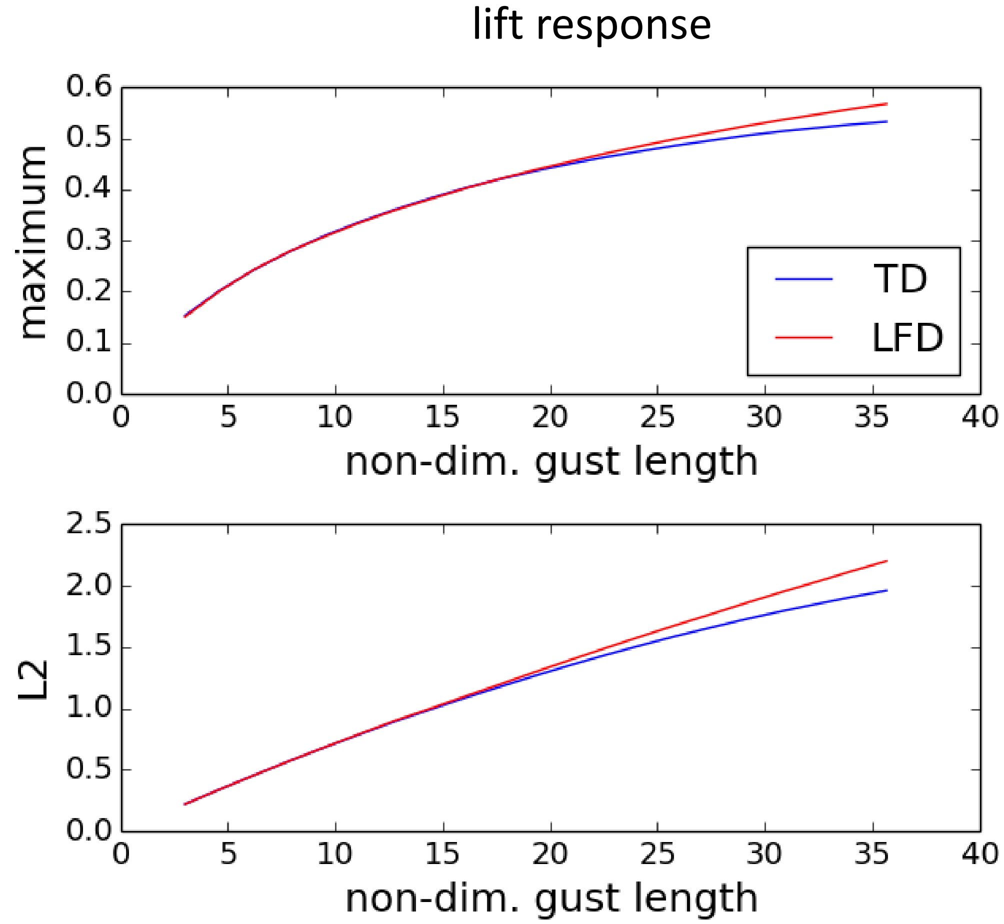
- CS 25: gust amplitude increases with gust length
- Linearised frequency domain (LFD) accurate for infinitesimally small amplitudes
- Impact on accuracy considering certifaction amplitudes ?



Frequency-Domain Non-linear Gust Response Computation

Motivation

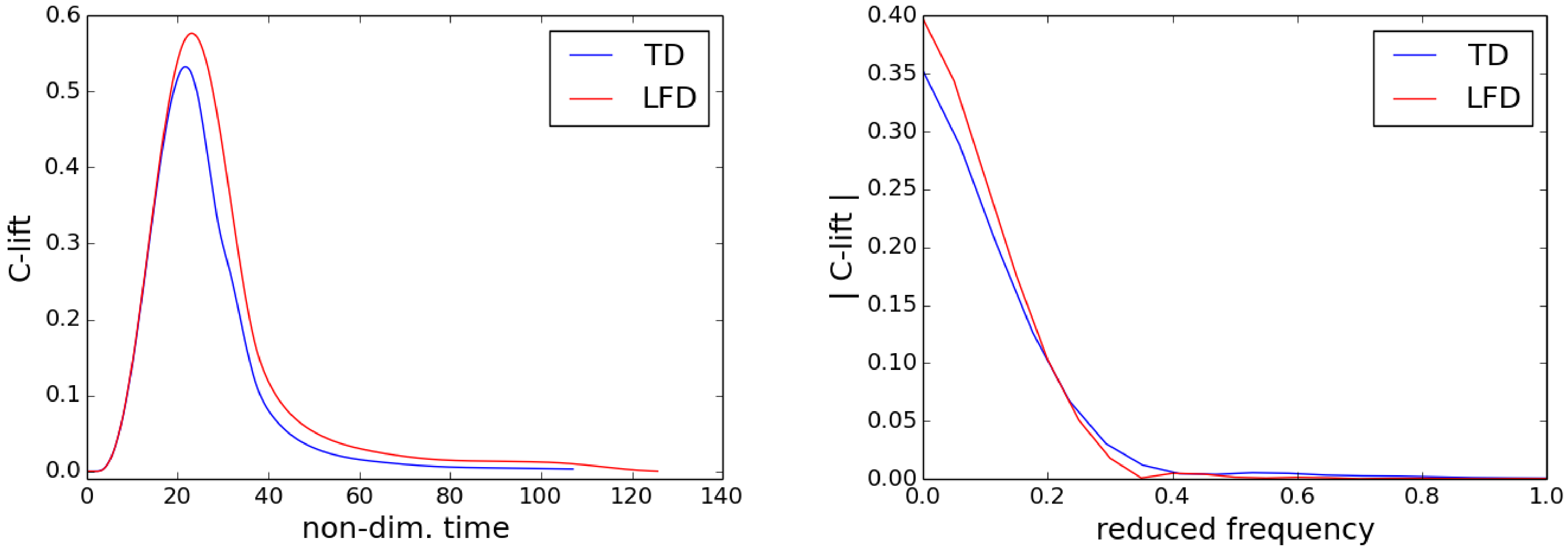
- CS 25: gust amplitude increases with gust length
- Linearised frequency domain (LFD) accurate for infinitesimally small amplitudes
- Impact on accuracy considering certification amplitudes?
  - Compare LFD to non-linear time-domain simulations
  - Shown is max. lift response and $\int_0^T \Delta c_L^2$ for NACA0012 test case
  - Good agreement till non-dim. gust length 20 (for a typical aircraft case: about 120m)



Frequency-Domain Non-linear Gust Response Computation

Motivation

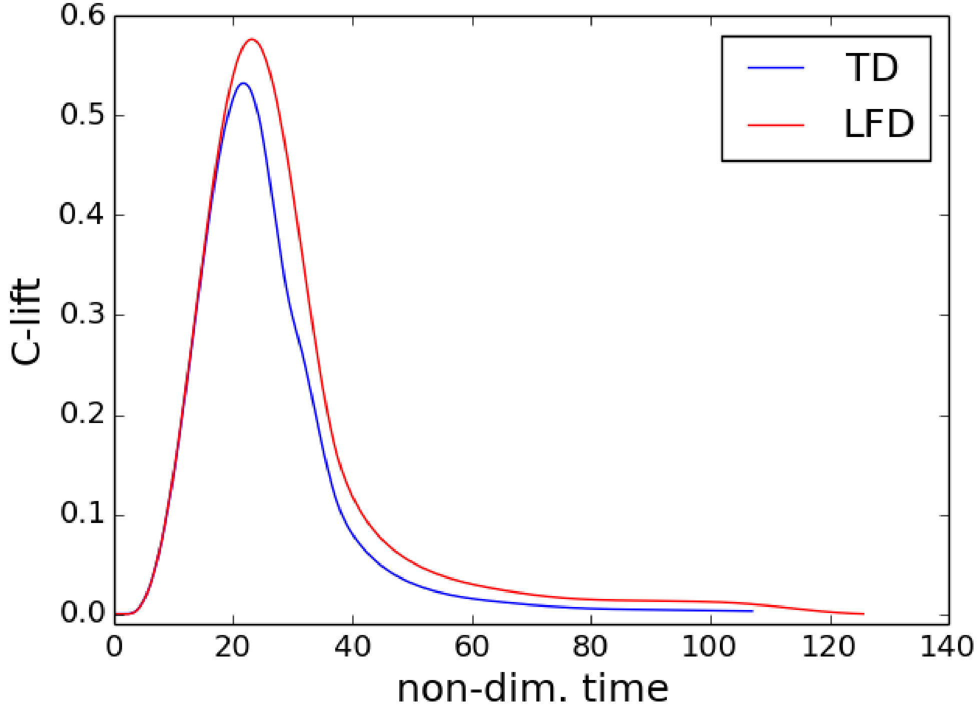
- Detailed analysis of largest gust length of 35

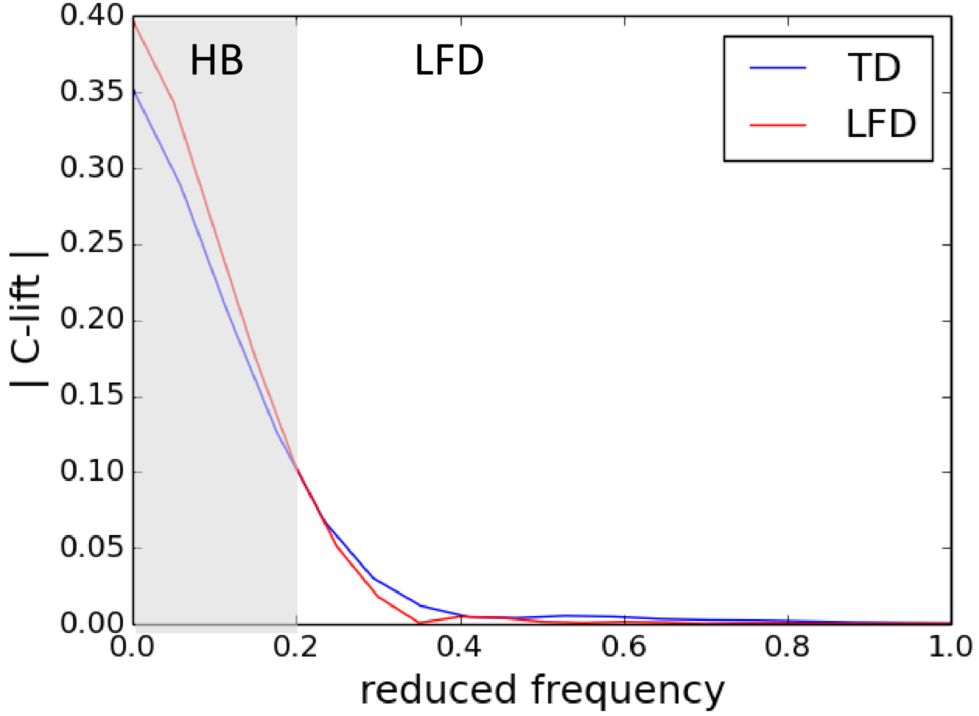


Frequency-Domain Non-linear Gust Response Computation

Idea

- Use Harmonic Balance (HB) method to enhance accuracy at low frequencies

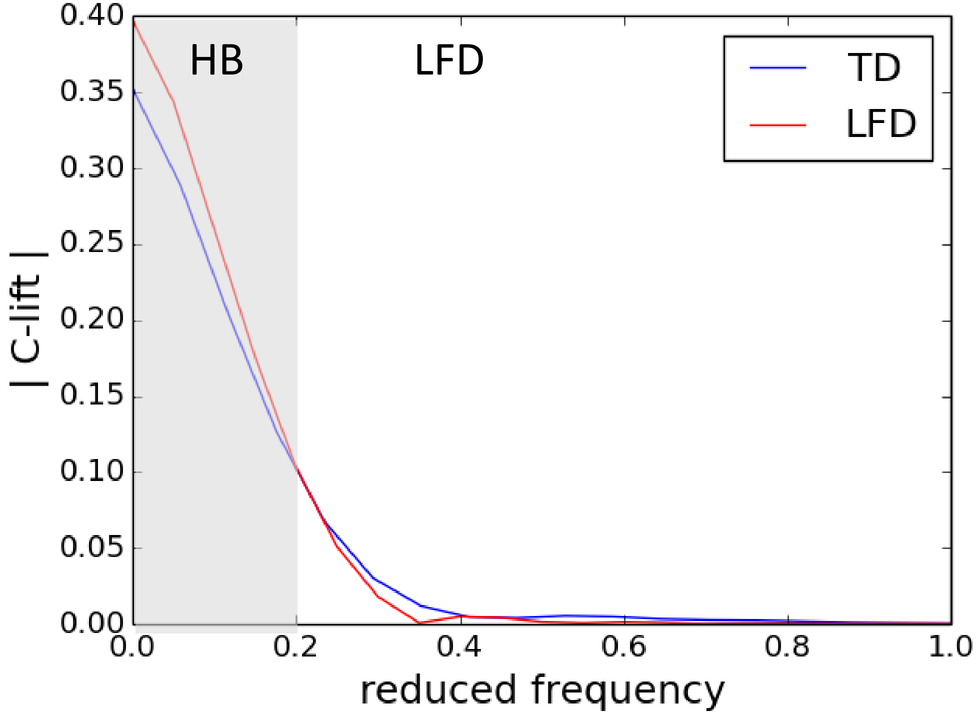




Frequency-Domain Non-linear Gust Response Computation

Idea

- Use Harmonic Balance (HB) method to enhance accuracy at low frequencies
- Not an amplitude non-linearity ”per frequency” (minor effect)
- Reduction in magnitude due to coupling between the harmonics of excitation and response
  → HB must be used for 1-cos gust,
  not single frequency sinusoidal
- Observation made for an aerofoil, but can we see a similar result for an aircraft case?

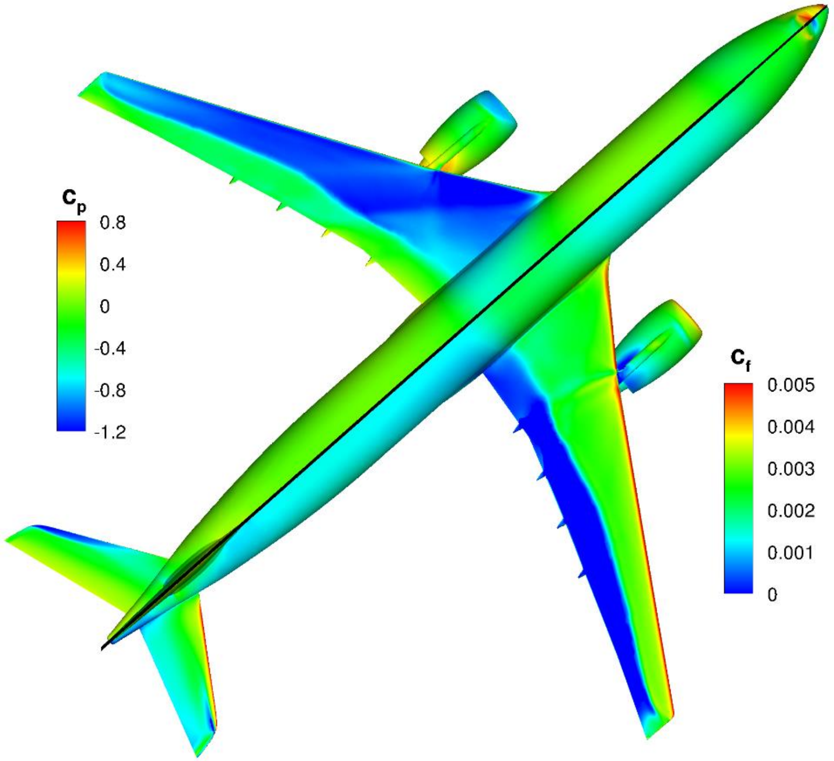


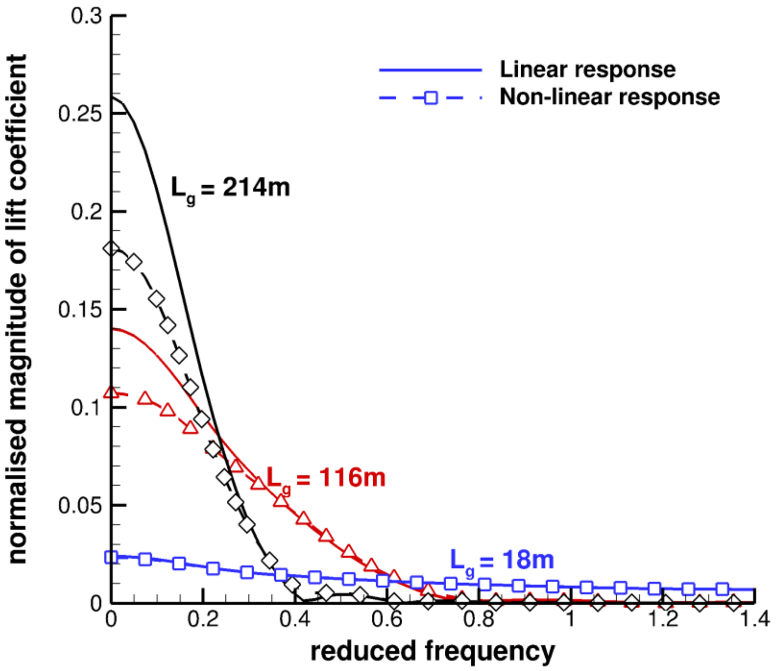
Frequency-Domain Non-linear Gust Response Computation

Idea

same for full aircraft case at cruise flight

Instantaneous at c_L-max





Frequency-Domain Non-linear Gust Response Computation

Approach

1. Calculate steady-state solution
2. Compute LFD solutions covering the relevant frequency range
3. Reconstruct time-domain response for small and medium gust lengths
4. For each “non-linear” gust length:
    1. Choose a base frequency and number of harmonics for Harmonic Balance method
    2. Solve HB equation
    3. Add LFD solutions for frequencies that are not covered by HB
    4. Reconstruct time-domain response

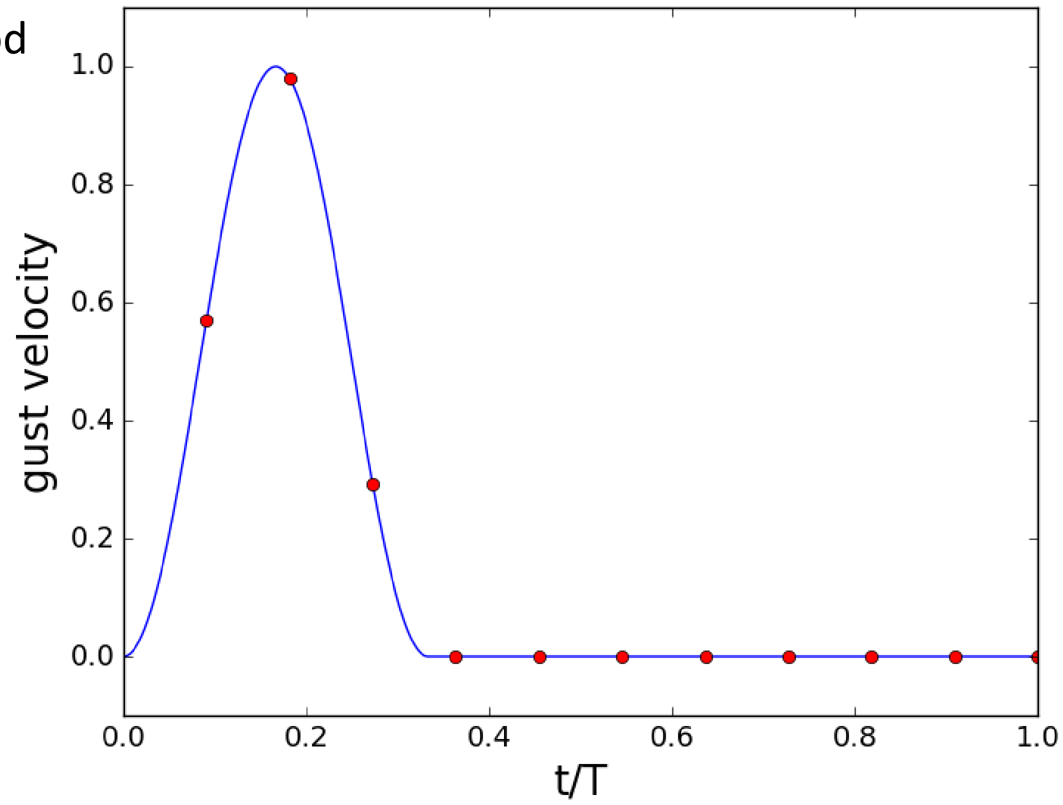
Frequency-Domain Non-linear Gust Response Computation

HB Approach

1. $2N_H + 1$ solution vectors equidistantly distributed over a period
2. Compute at each time-slice the residual vector
3. Transform into frequency-domain
4. Compute update via pseudo-time integration

$$\frac{dW_{HB}}{d\,\tau} = \omega_b D W_{HB} + R_{HB}$$

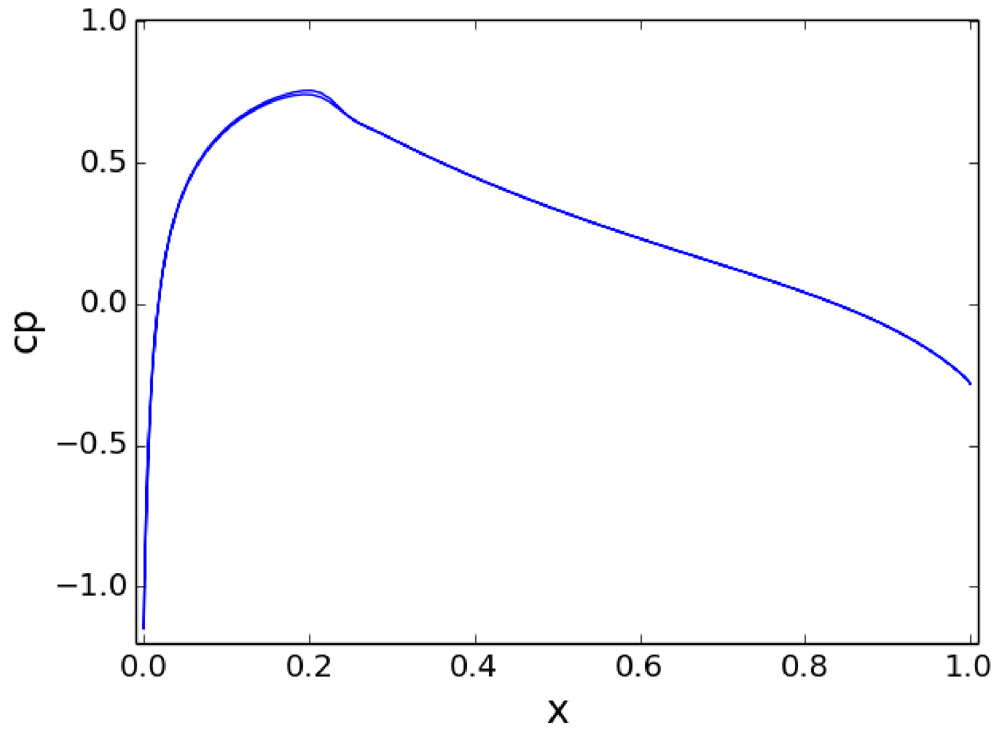
$$D_{ik} = \frac{2}{2N_H - 1} \sum_{m=1}^{N_H} m \sin \frac{2\pi(k-i)m}{2N_H + 1}$$



Frequency-Domain Non-linear Gust Response Computation

Results: NACA0012

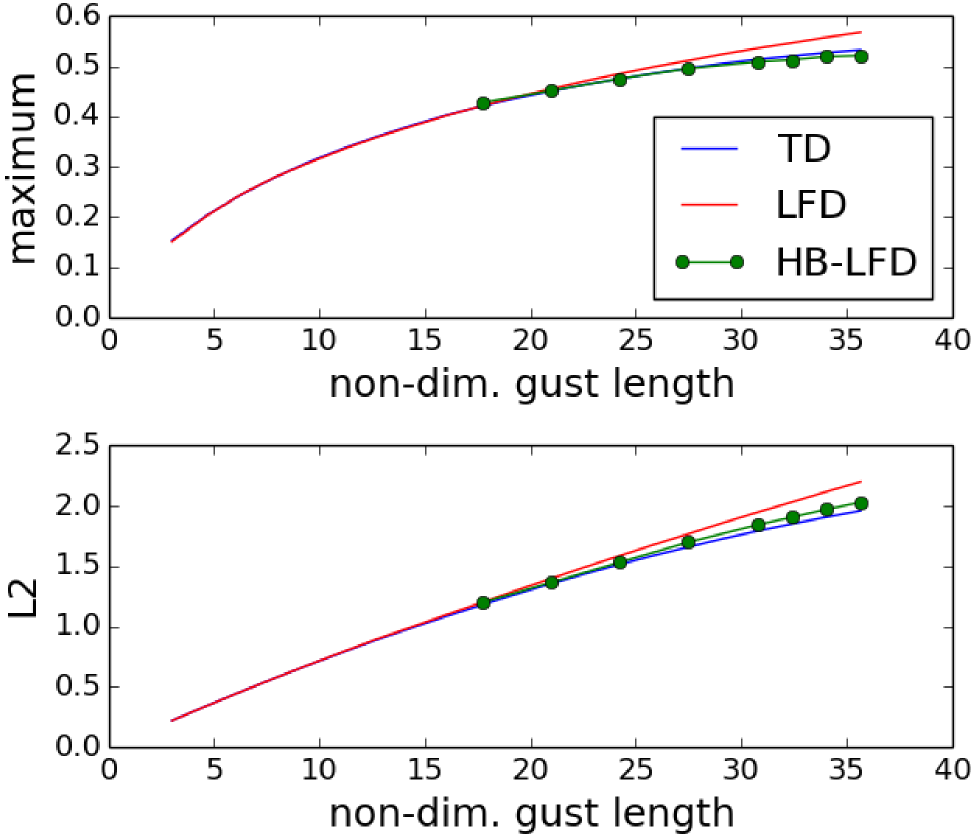
- Mach 0.75, AoA = 0 deg., Re = 10 million
- Weak transonic case



Frequency-Domain Non-linear Gust Response Computation

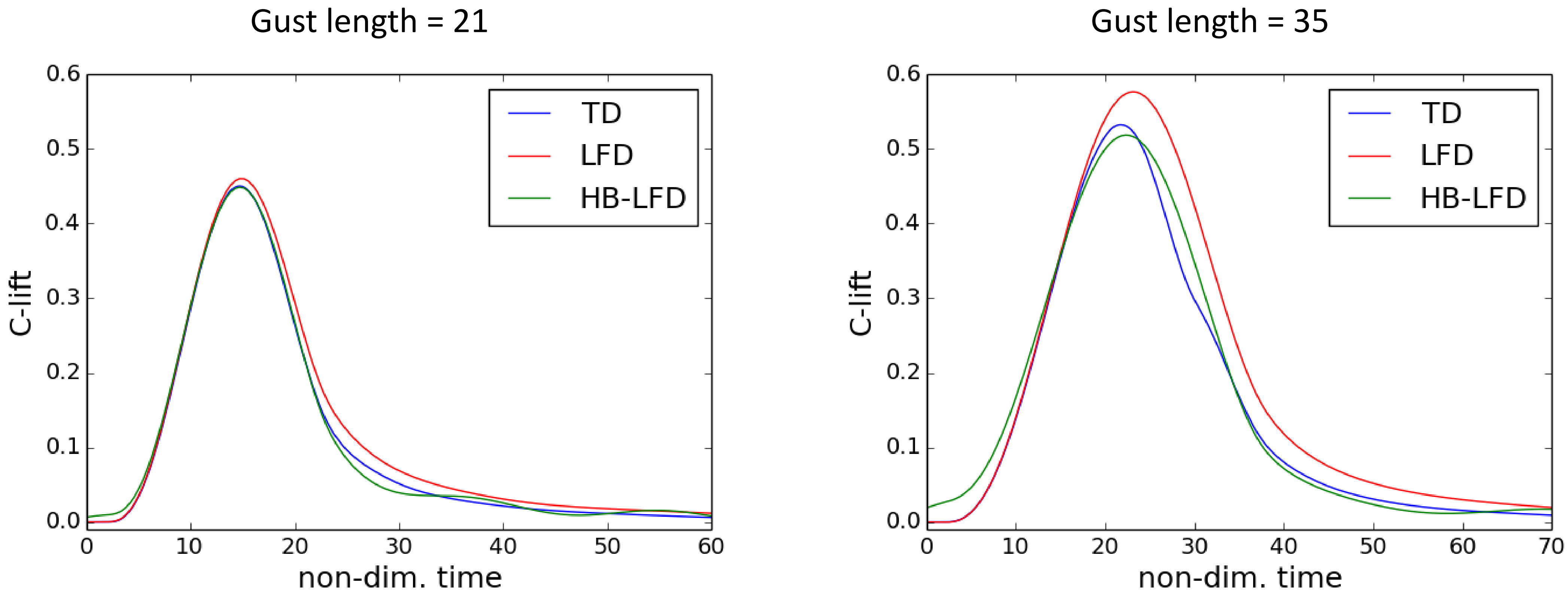
Results: NACA0012 – HB-LFD with 3 harmonics

- Mach 0.75, AoA = 0 deg., Re = 10 million
- Weak transonic case
- Harmonic Balance with 3 harmonics
- Significant improvement in both norms
- Small deviations remain at highest gust lengths
- about 5x faster then TD per gust simulation



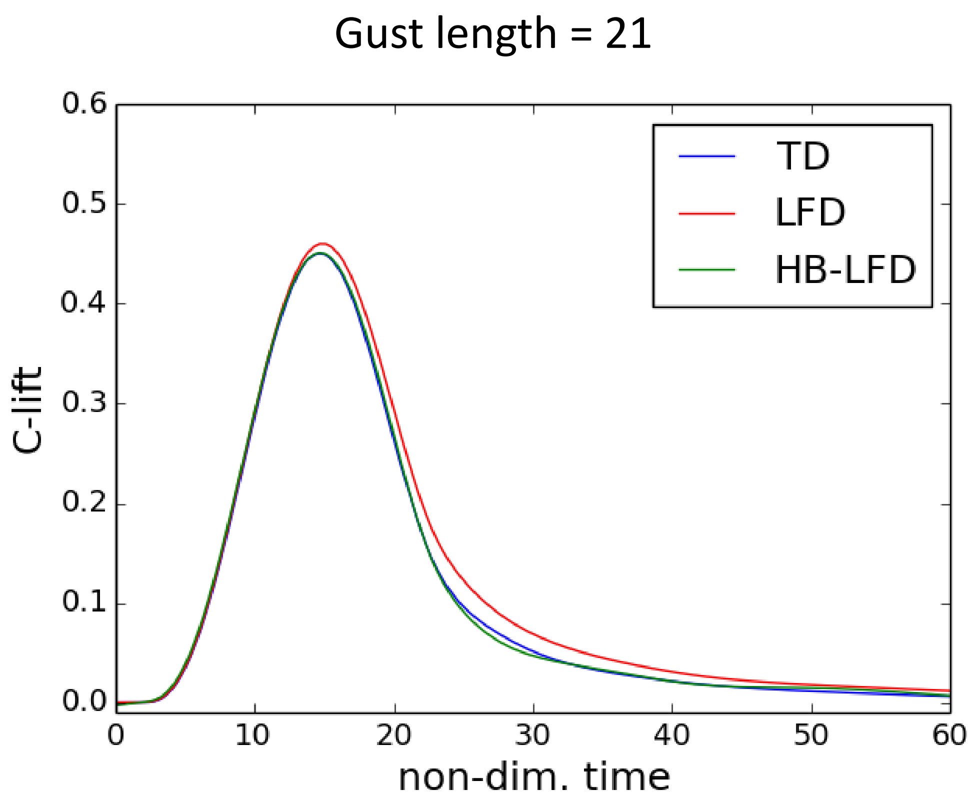
Frequency-Domain Non-linear Gust Response Computation

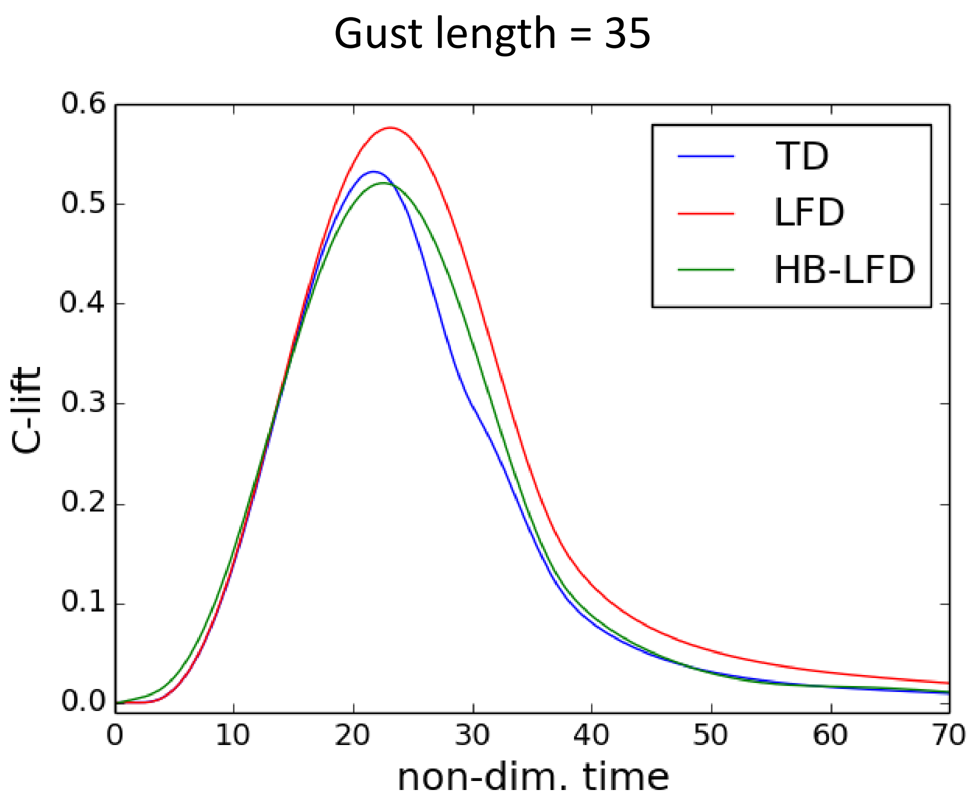
Results: NACA0012 – HB-LFD with 3 harmonics



Frequency-Domain Non-linear Gust Response Computation

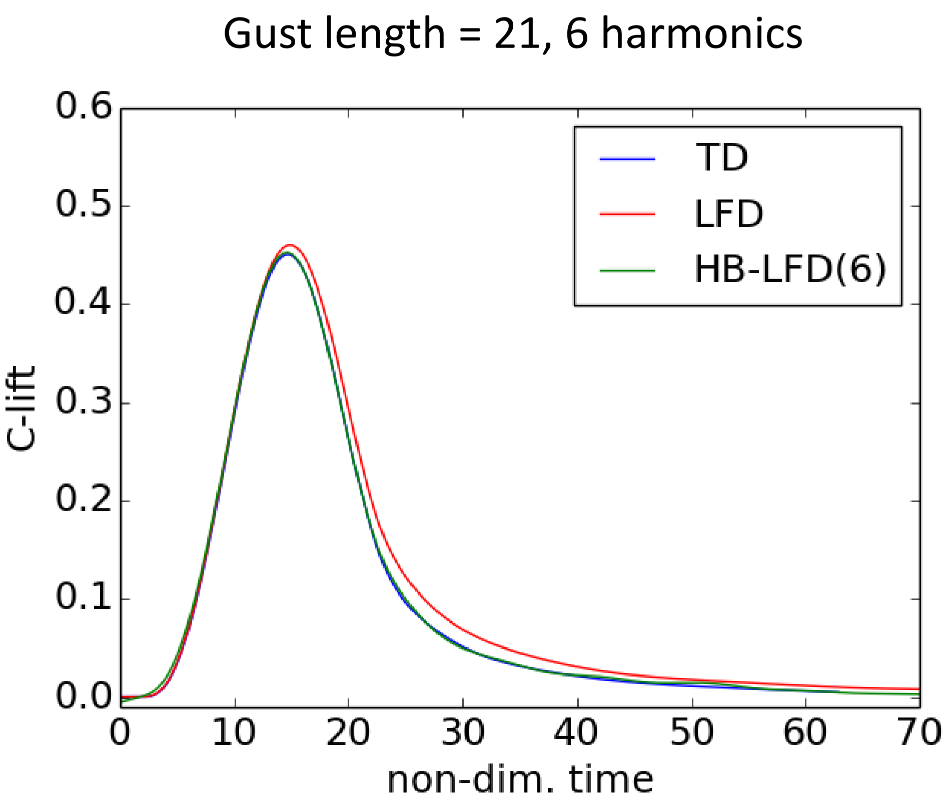
Results: NACA0012 – HB-LFD with 4 harmonics

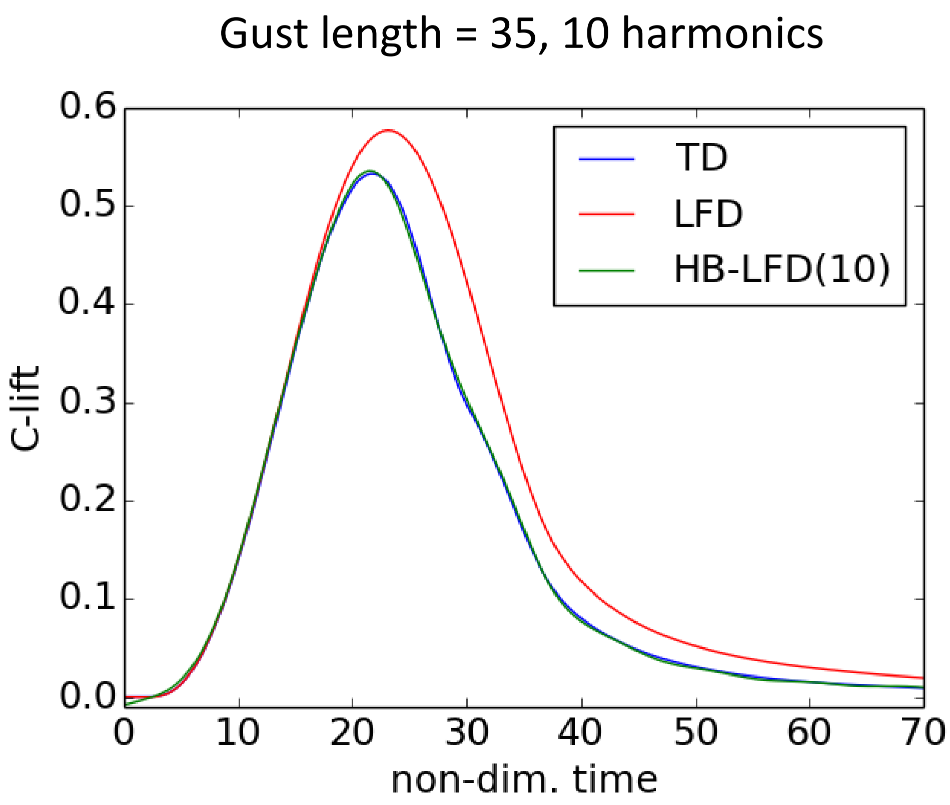




Frequency-Domain Non-linear Gust Response Computation

Results: NACA0012 – HB-LFD best fit





Intermediate conclusion

- Aerodynamic responses of gust encounter compared between linearised frequency domain and non-linear time-domain simulations using CS-25 gust definitions
  - Good agreement for small and medium gust lengths for NACA0012 aerofoil
  - Lift response over-estimated by LFD for larger gust lengths and amplitudes
- Applying Harmonic Balance method with a small number of harmonics combined with LFD results for higher frequencies yields improvement for NACA0012

Next step:

- Compute gust response of fluid-structure coupled configuration using Harmonic Balance and LFD

LFD4Gust with FSI

- Rearrange structural equation in system of 1st order ODE
- Augmented LFD system

$$\left(\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} - j\omega I\right)\begin{bmatrix} w_f \\ w_s \end{bmatrix} = \begin{bmatrix} b_f \\ 0 \end{bmatrix}$$

with subscripts *f* and *s* denoting fluid or structural DoF, respectively

- Right-hand-side vector defined by field-velocity method

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HB4Gust with FSI

- Similar to LFD, the system of equations and the vector of unknowns is augmented with their structural part
- Thus, HB solves for W_f and W_s at each time slice
- Corresponding fluid and structural residuals are computed
  - Involves updating grid point locations and velocities according to structural motion for each time slice
  - Grid movement can be realised using deformation or here rigid-body motion (pitch-plunge aerofoil)
- The rest is usual HB approach
- For implicit solution scheme, coupled Jacobians are used (see LFD solver)

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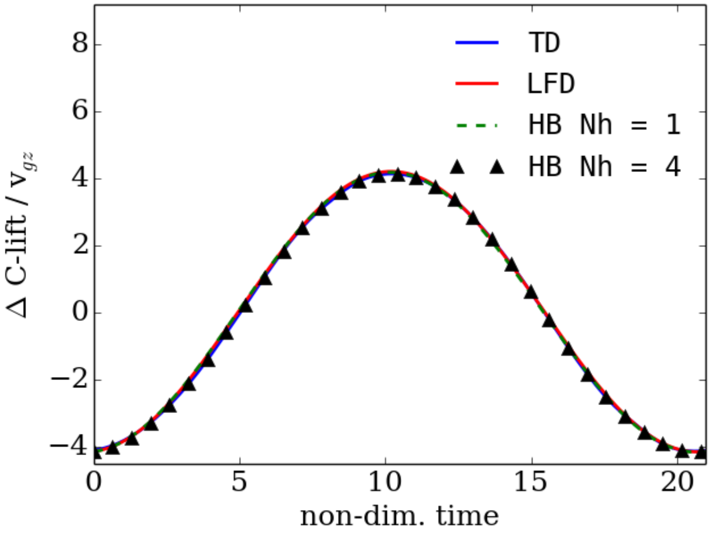
HB4Gust with FSI

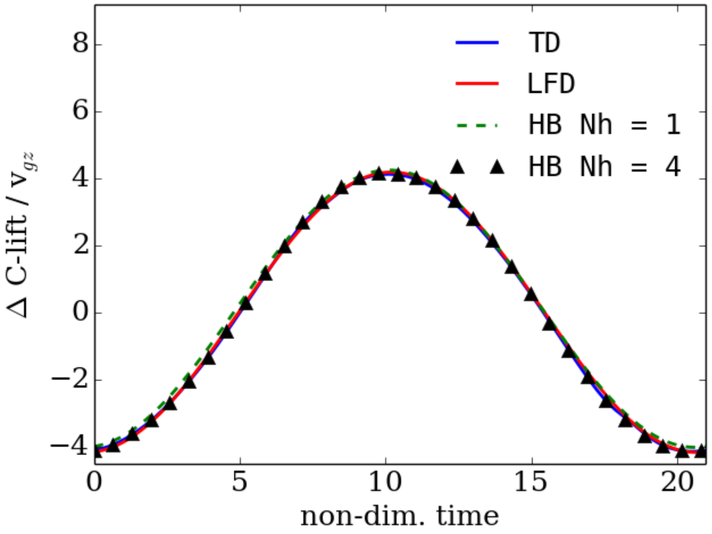
- Previous test case extended by pitch-plunge structure
- In-vacuum reduced frequencies of 0.34 for heave and 1.0 for pitch
- Sinusoidal gust encounter with wave length of 21 chord lengths and two gust amplitudes
- TD Signal recorded after 20 periods

Lift response

v_{gz} = 6% free-stream velocity



v_{gz} = 12% free-stream velocity



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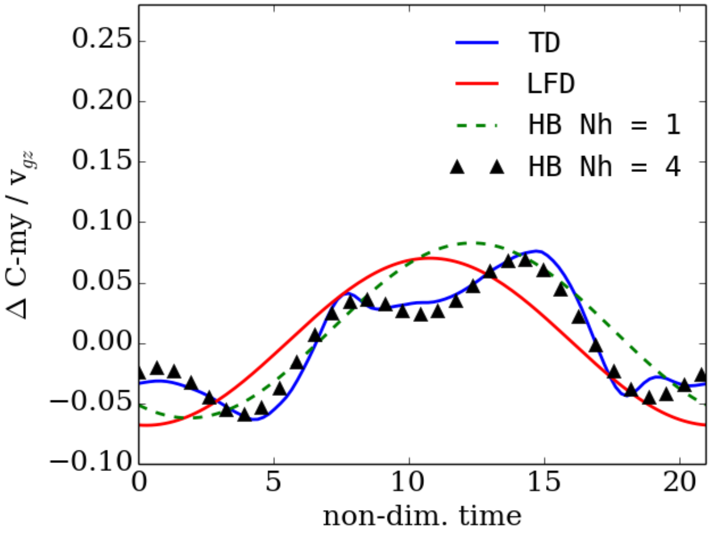
Moment response

v_{gz} = 6% free-stream velocity



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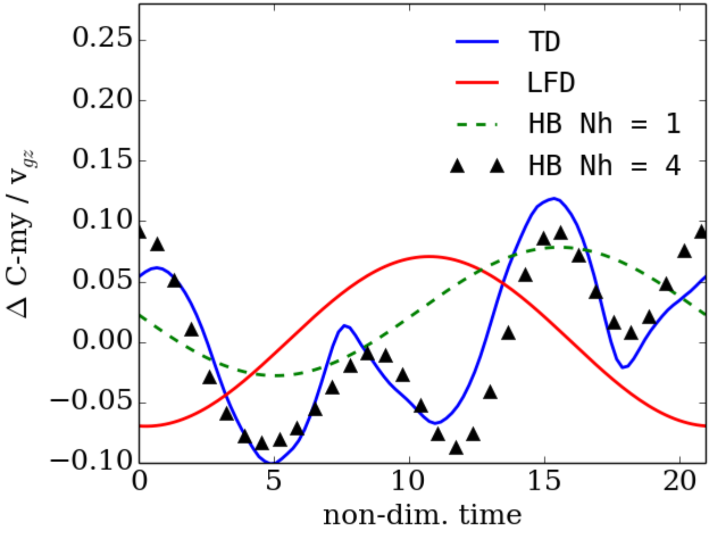

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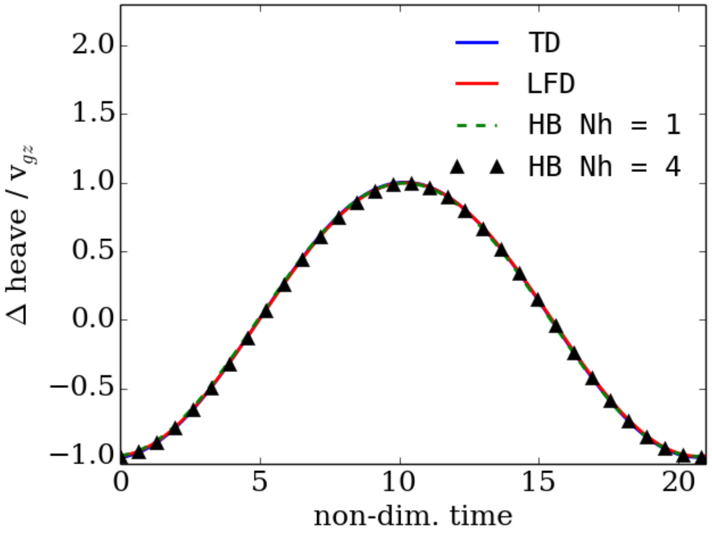
Heave response

v_{gz} = 6% free-stream velocity



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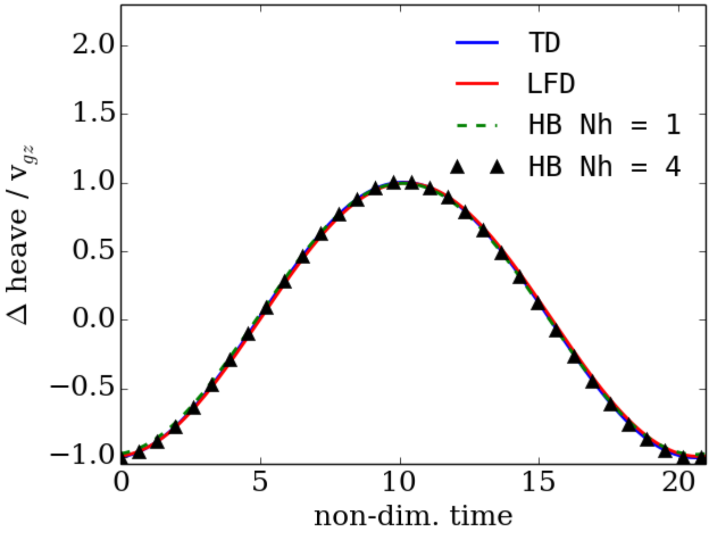

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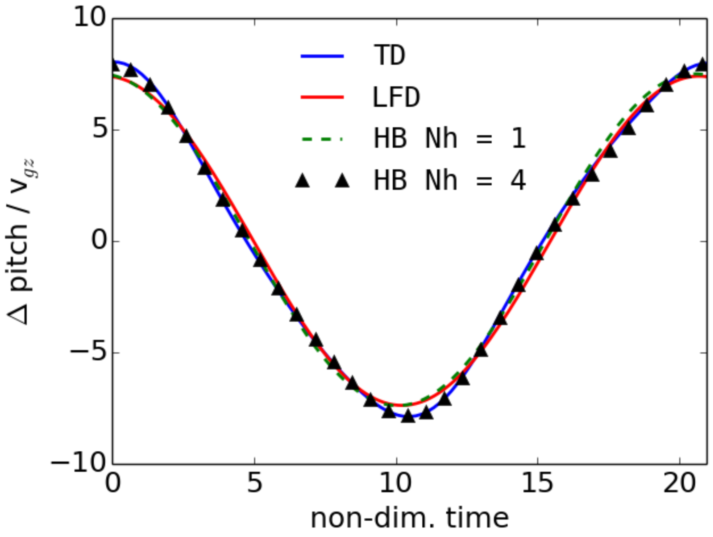
Pitch response

v_{gz} = 6% free-stream velocity



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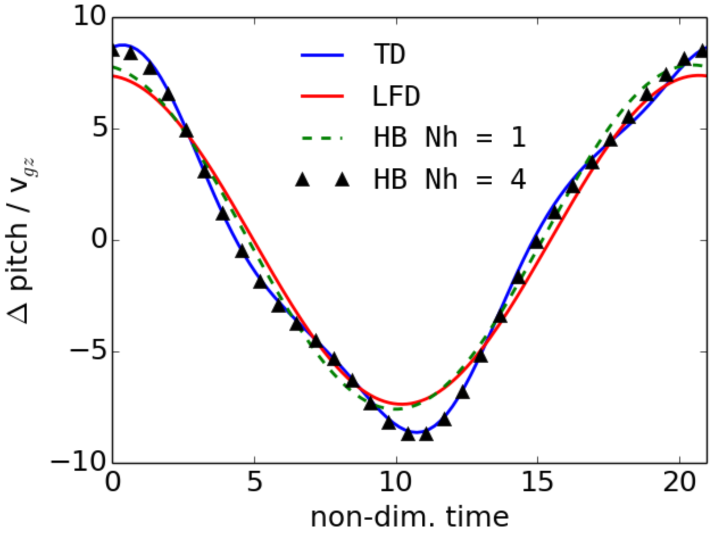


Conclusion

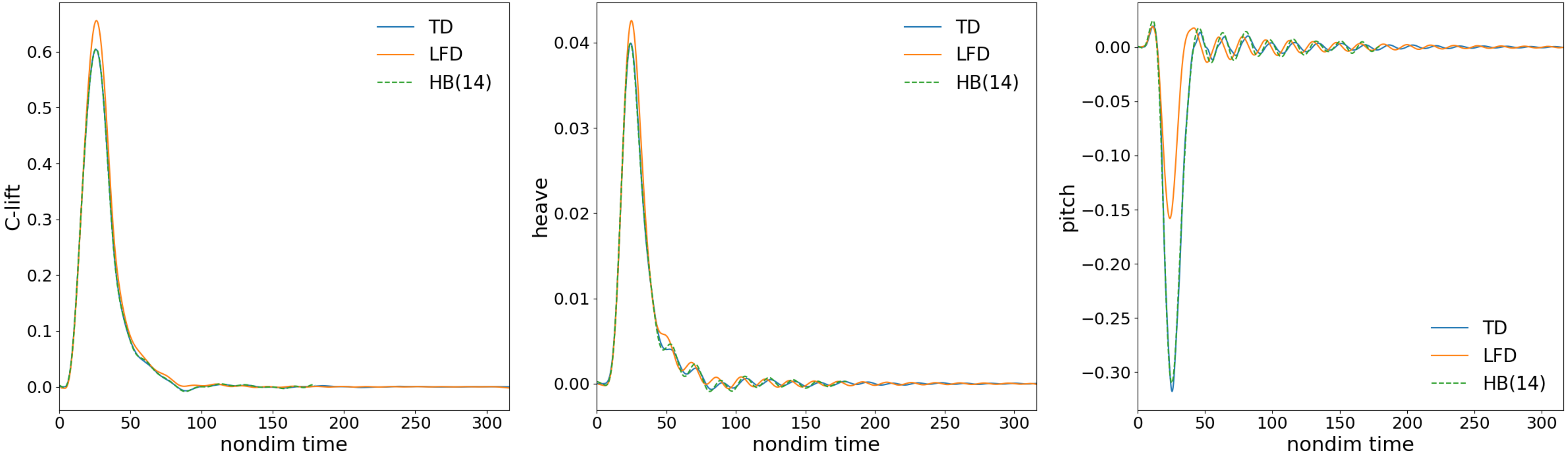
- LFD and HB solver extended to compute response of fluid-structure coupled systems due to gust encounter
- Demonstrated for sinusoidal gusts
  - Good agreement between HB(4) and TD reference
  - Lift and heave response dynamically linear → LFD sufficient
  - Contributions of higher harmonics for moment and pitch response
  - Nonlinearities captured well by HB method

Future steps:

- Application to 1-cos gusts
- Apply symbiotic approach of HB-LFD to coupled system

First results: 1-cos gust

- 1-cos response of longest gust: Lg = 35.5, v_{gz} = 6.6%
- Lift and heave response is over-predicted by LFD while pitch response is under-predicted
- HB(14)(!) improves the prediction at the peak



Thank you!

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