# A COMPARISON OF LINEAR AND NON-LINEAR FLUTTER PREDICITON METHODS: A SUMMARY OF PUMA DARP AEROELASTIC RESULTS

C.B.Allen, D.Jones and N.V.Taylor Department of Aerospace Engineering, University of Bristol, Bristol, BS8 1TR, U.K.

K.J.Badcock, M.A. Woodgate and A.M.Rampurawala Department of Aerospace Engineering,

University of Glasgow, Glasgow G12 8QQ, U.K.

J.E. Cooper and G.A.Vio School of Engineering, University of Manchester, Manchester, M13 9PL, U.K.

August 26, 2004

## Abstract

This paper presents a comparison of linear and nonlinear methods for the analysis of aeroelastic behaviour and flutter boundary prediction. The methods in question include NASTRAN, ZAERO, and the coupled CFD-CSD methods RANSMB and PMB, developed at the Universities of Bristol and Glasgow respectively. The test cases used for the comparison are the MDO and AGARD 445.6 weakened wing. In general, it was found that the non-linear methods demonstrate excellent agreement with respect to pressure distributions, deflections, dynamic behaviour, and flutter boundary locations for both cases. This is in contrast to previous studies involving similar methods, where notable differences across the MDO span were found, and is taken to imply good performance of the interpolation schemes employed here. Whilst the linear methods produce similar flutter boundaries to the coupled codes for the aerodynamically simple AGARD 445.6 wing, results for the transonic 'rooftop' MDO wing design were in less close agreement.

# Introduction

The prediction of flutter is an important area of aircraft design. Methods currently common in industry for this purpose involve the use of linear techniques, allowing uncoupling of the aerodynamic and structural equations. However, it has long been recognised that this reduces the accuracy of the methods, as real flows of practical interest are often non-linear in nature (particularly transonic flight). As flutter must be avoided at all costs, this leads to the need for significant safety margins, creating over-stiff and hence high mass designs.

Prediction methods proven to be of greater accuracy will therefore lead directly to weight savings, without requiring any advances in the underlying aerodynamic or structural design methods. For this reason, non-linear techniques are under widespread development, consisting of time-accurate CFD analysis of the flow coupled to a dynamic structural model. However, such techniques are considerably more time consuming and computationally expensive, and the question naturally arises as to when and where these methods are required, and when the simpler linear methods provide adequate solutions. This paper addresses this issue by a comparison of the performance of two coupled CFD-CSD codes (PMB and RANSMB), and two commercially available linear methods, NASTRAN and ZAERO, for a pair of widely known aeroelastic test cases, specifically the AGARD 445.6 weakened wing (wing 3), and the MDO wing.

## Description of CFD Codes

# Aerodynamic Formulation and Solution

Although both PMB and RANSMB are capable of Navier-Stokes solutions, such are currently prohibitively time consuming for three dimensional unsteady analysis. For this reason, only the Euler equations were solved. These may be written in nondimensional conservative form as

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \frac{\partial \mathbf{H}}{\partial z} = 0 \tag{1}$$

where  $\mathbf{W} = (\rho, \rho u, \rho v, \rho w, \rho E)^T$  denotes the vector of conservative variables. The flux vectors **F**, **G** and **H** are,

$$\mathbf{F} = \begin{pmatrix} \rho U \\ \rho u U + p \\ \rho v U \\ \rho w U \\ U(\rho E + p) + x_t p \end{pmatrix}$$
$$\mathbf{G} = \begin{pmatrix} \rho V \\ \rho u V \\ \rho v V + p \\ \rho w V \\ V(\rho E + p) + y_t p \end{pmatrix}$$
$$\mathbf{H} = \begin{pmatrix} \rho W \\ \rho u W + p \\ \rho v W \\ \rho w W + p \\ W(\rho E + p) + z_t p \end{pmatrix}$$

In the above  $\rho$ , u, v, w, p and E denote the density, the three Cartesian components of the velocity, pressure and the specific total energy respectively. The terms U, V and W are the contravariant velocities defined by

$$U = u - x_t, \ V = v - y_t, \ W = w - z_t, \tag{2}$$

where  $x_t$ ,  $y_t$  and  $z_t$  are the grid speeds in the Cartesian directions.

The Euler equations are discretised using the two codes described below on curvilinear multi-block body conforming grids using a cell-centred finite volume method which converts the partial differential equations of (1) into a set of ordinary differential equations which can be written as

$$\frac{d}{dt}(V_{i,j,k}\mathbf{W}_{i,j,k}) = -\mathbf{R}_{i,j,k}(\mathbf{W}).$$
(3)

Solution in the University of Glasgow code PMB (Parallel MultiBlock) code is through an implicit method based on Osher's [5] upwind method. MUSCL variable extrapolation [6] is used to provide second-order accuracy with the Van Albada limiter to prevent spurious oscillations around shock waves. The time derivative is approximated by a second-order backwards derivative [7]. The University of Bristol has extended the BAE SYSTEMS code RANSMB (Reynolds Averaged Navier-Stokes MultiBlock), a cell centred finite volume solver of a Jameson type [8], to allow fully implicit unsteady calculations through Pseudo-time stepping [7, 9]. Convergence in the time domain occurs through a modified Runga-Kutta scheme due to Melson [10].

# Structural Modelling

In all cases reported here the structural behaviour was modelled using a modal approximation derived from the comercial FE package MSC NASTRAN. Mode shapes and frequencies are derived by solving the equation

$$[\mathbf{M} - \omega_i^2 \mathbf{K}]\phi_i = \mathbf{0} \tag{4}$$

where  $\phi_i$  are the mode shape vectors and  $\omega_i$  the frequencies.

Introducing an arbitrary level of damping  $[\mathbf{C}]$ , defined following Ref. 11 by

$$[\mathbf{C}] = a[\mathbf{K}] + b[\mathbf{M}] \tag{5}$$

into the system allows more rapid attainment of steady state deflections under stable flow conditions, as well as allowing steady state responses beyond the flutter boundary to be determined. It also allows analysis of the actual effects of realistic structural damping on models, where damping behaviour is known. The equation of motion of the structure may then be expressed as

$$[\mathbf{M}]\delta\ddot{\mathbf{x}}_s + [\mathbf{C}]\delta\dot{\mathbf{x}}_s + [\mathbf{K}]\delta\mathbf{x}_s = \mathbf{f}_s \tag{6}$$

where  $\delta \mathbf{x}_s$  is a vector of displacements on a grid of points  $\mathbf{x}_s$ , and  $\mathbf{f}_s$  is the external force. If the structural

motion is assumed linear (and hence  $[\mathbf{M}], [\mathbf{C}], \text{ and } [\mathbf{K}]$  and then by the principle of virtual work,  $\mathbf{f}_s = S^T \mathbf{f}_a$ . are constant for a given structure), this equation may be solved either by modal analysis, or by direct integration of  $[\mathbf{M}]$ ,  $[\mathbf{C}]$ , and  $[\mathbf{K}]$ .

Modal Equations The modal output is nondimensionalised with respect to mass such that

$$\phi_i^T[\mathbf{M}]\phi_i = 1 \tag{7}$$

Using this approach results in a modal version of equation (6):

$$\ddot{\alpha} + 2\omega\zeta\dot{\alpha} + \omega^2\alpha = [\phi]\mathbf{f}_s \tag{8}$$

where  $\omega$  is the vector of mode frequency,  $\zeta$  that of damping ratio,  $\alpha$  the vector of modal displacements, and  $[\phi]$  the matrix composed of the mode shape vectors  $\phi$ . The value of the damping ratio follows from the specification of a and b in equation (5) via

$$\zeta_i = \frac{1}{2} \left( a\omega_i + \frac{b}{\omega_i} \right)$$

and may either be selected to best represent the damping behaviour of a real wing or, if artificial damping is intended,  $\zeta_i$  itself may be directly specified to give the desired characetristics. Once the modal displacement is calculated, the Cartesian displacement at each node follows from

$$\delta \mathbf{x}_s = [\phi] \alpha \tag{9}$$

These equations are solved using a Runga-Kutta approach in PMB, and a Newmark Scheme in RANSMB [12]. Strong coupling is used to avoid introducing sequencing errors and phase lags. In both codes, the formulation is such that time step size may be selected on the basis of the model dynamics rather than any CFD or coupling considerations.

## Inter-Grid Transfer

The aerodynamic forces are calculated at face centres on the aerodynamic surface grid. The problem of communicating these forces to the structural grid and returning the structural deflections to the fluid surface grid is complicated in the common situation that these grids not only do not match, but are also not even defined on the same surface. Denoting the fluid surface grid locations and aerodynamic forces as  $\mathbf{x}_a$  and  $\mathbf{f}_a$ , then a linear relationship to the corresponding structural values is used in the form

$$\delta \mathbf{x}_a = S(\mathbf{x}_a, \mathbf{x}_s) \delta \mathbf{x}_s \tag{10}$$

The matrix S is called the spline matrix.

The grid speeds on the wing surface are also needed and these are approximated directly from the transformation as

$$\delta \dot{\mathbf{x}}_a = S(\mathbf{x}_a, \mathbf{x}_s) \delta \dot{\mathbf{x}}_s$$

where the structural grid speeds are given by

$$\delta \dot{\mathbf{x}}_s = \Sigma \dot{\alpha}_i \phi_i. \tag{11}$$

Both solvers make use of the CVT (Constant Volume Tetrahedra) approach due to Goura [13], in linear form. In PMB the each aerodynamic point is assigned a tetrahedra, and the tetrahedra equations are relinearised at each time step. RANSMB makes use of an intermediate plane formed by the intersection of the outward normals of the structural grid and the aerodyanmic surface. The motion of this intermediate grid is derived through consideration of CVT's formed between it and the structural mesh, and the TPS (Thin Plate Spline) method is then used to interpolate from the intermediate grid to the aerodynamic surface [14].

Volume Grid Movement Both codes interpolate volume grid points  $\mathbf{x}_{ijk}$  as

$$\delta \mathbf{x}_{ijk} = \psi_j^0 \delta \mathbf{x}_{a,ik} \tag{12}$$

where  $\psi_i^0$  are values of a blending function [15] which varies between one at the wing surface (here j=1) and zero at the block face opposite. The surface deflections  $\mathbf{x}_{a,ik}$  are obtained from the transformation of the deflections on the structural grid and so ultimately depend on the values of  $\alpha_i$ . In PMB block sides internal to the flow or on other types of boundaries are fixed, whereas RANSMB employs a sprung block system.

Cell volumes are recalculated using a global conservation law by considering volume fluxes through cell sides. In the present calculations the geometric conservation law (GCL) was used. The input for the grid movement method is therefore the change in the point locations and velocities on the wing surface from the last time the volume grid vertex locations and speeds were calculated. This information is obtained from the structural solution through the transformation defined above.

#### Description of Linear Methods

The aeroelastic analysis has been undertaken using the commercially available NASTRAN and ZAERO packages. Of the various modules available in NASTRAN, it is the DLM code that is of primary use for the transonic analysis considered here. The ZAERO package contains five separate modules for different flow conditions from subsonic to hypersonic. Those used here are ZONA6, based on linear subsonic unsteady aerodynamics, and ZTAIC, a non-linear unsteady transonic aerodynamic solver. These methods use a frequency domain approach for flutter calculations, requiring generation of large UAIC (Unsteady Aerodynamic Influence Coefficient) matrices to compute the appropriate aerodynamics.

The doublet-lattice method (DLM) [16-18] implemented in both NASTRAN and ZAERO may be used for the analysis of interfering lifting surfaces in subsonic flow. It is an extension of the steady vortex-lattice method to unsteady flow, and is based on linearised aerodynamic potential theory. The undisturbed flow is uniform and is either steady or varying harmonically. All lifting surfaces are assumed to lie nearly parallel to the (subsonic) flow. Each of the interfering surfaces is divided into small trapezoidal lifting elements such that the elements are arranged in strips parallel to the free stream with surface edges, fold lines and hinge lines lying on the element boundaries. The unknown lifting pressures are assumed to be concentrated uniformly across the one-quarter chord line of each element. There is one control point per element, centred spanwise on its three-quarter chord line, and the surface downwash boundary condition is satisfied at each of these points. For supersonic flows, another linearised method is used (ZONA51), which has similar assumptions (no thickness effects, small angles, etc.).

The main difference between the linear ZONA6 and NASTRAN'S DLM is that the former applies to the complete aircraft and not just to the lifting surfaces. Thus the aerodynamic effect of the fuselage and/or external stores may be incorporated in a ZAERO analysis. However, in the present analysis of isolated wings, this feature is irrelevent.

ZAERO's ZTAIC module is unique among commercial aeroelastic packages. This module requires external input in the form of steady pressure at various spanwise positions. Zonatech state that experimental data is usually preferable, but this is often unavailable, whereas the CFD approach is relatively simple. In this case the steady surface pressure data was generated by Glasgow University, using the PMB3D code. This information is used to update the AIC matrix, and allows for better approximation of the flutter boundary in the transonic flow region.

It is well known that the transonic small disturbance theory may not provide accurate solutions for strong transonic shock cases because it cannot correctly model the entropy gradient from strong shock waves, nor convect the vorticity. However this is not to say that the transonic small disturbance theory is not suitable for the prediction of unsteady flows due to small aeroelastic deformations if the total unsteady flow is decomposed into a steady background flow and an unsteady component made up of small disturbances. Simplified theories based on the small disturbances approach can yield accurate unsteady flow predictions [19], provided that steady background flow on which the unsteady disturbance propagate is accurately accounted for.

The ZTAIC method performs an inverse airfoil design that generates an airfoil surface based on the input steady pressure. This design airfoil surface is used to generate the unsteady transonic pressure distribution.

#### Intergrid Splining

The ZAERO and NASTRAN packages couple the structural and aerodynamic model through a spline matrix (using Thin or Infinite Plate, Beam, or Rigid Body Attachement Spline methods), that gives all the displacements and slopes in the (x, y, z) directions at all aerodynamic panel control points. The interpolated matrix  $[G_{kg}]$  relates the component of structural grid point deflection  $\{u_g\}$  to the deflection of the aerodynamic grid point  $\{u_k\}$ 

$$\{u_k\} = [G_{kg}]\{u_g\}$$
(13)

This transformation ensures structural rather than static equivalence, i.e. the loads produce the same structural displacement. The aerodynamic forces  $\{F_k\}$ and their structural equivalent values  $\{F_g\}$  acting on the structural grid points therefore do the same virtual work in their respective deflection modes

$$\{\delta u_k\}^T \{F_k\} = \{\delta u_g\}^T \{F_g\}$$
(14)

where  $\delta u_k$  and  $\delta u_g$  are virtual deflections. Substituting Eqn. 13 into the left-hand side of Eqn. 14 and rearranging yields



Figure 1: Flutter Boundaries, AGARD 445.6 Wing

$$\{\delta u_g\}^T \left( [G_{kg}]^T \{F_k\} - \{F_g\} \right) = 0$$
 (15)

from which the required force transformation

$$\{F_g\} = \left[G_{kg}\right]^T \{F_k\} \tag{16}$$

is obtained because of the arbitrariness of the virtual deflections. Eqs. 13 and 16 are both required to complete the formulation of aeroelastic problems in which the aerodynamic and structural grid points do not coincide.

#### Dynamic Test Cases

# AGARD 445.6 Weakened Wing

The AGARD 445.6 wing is made of mahogany and has a 45° quarter chord sweep, a root chord of 22.96 inches and a constant NACA64A004 symmetric profile [20]. A series of flutter tests, carried out at the NASA Langley Transonic Dynamics Tunnel to determine stability characteristics, were reported in 1963. Of the various wing models tested, the most published results appear for the weakened wing (wing 3) in air. This wing had holes drilled out and filled with plastic to reduce stiffness whilst maintaining aerodynamic shape. Published experimental data includes the flutter boundary for several Mach numbers in the range 0.338 to 1.141. The structural characteristics of the wing were provided in the form of measured natural frequencies and mode shapes derived from a finite element model.

The wing has been the subject of many previous studies, using linear [21], linear plus non-linear corrections [22], TSD [23], TSD plus boundary layer [24], and structured and unstructured Euler and Navier Stokes CFD [25–29] methods. Generally, it was found that the TSD results were less accurate than linear methods, that the CFD techniques were superior overall, but were poor in the supersonic region, and that grid refinement remained an issue. Viscosity and mode number in the structural model were found to have an effect, but the magnitude of this depended on Mach number.

A comparison of the flutter boundaries produced by the methods under investigation is shown in Figs. 1(a)and 1(b). The flutter boundaries predicted by the linear methods (Fig. 1(a)), produce the correct overall trends, and locate the transonic dip at the same Mach number as the experimental results. However, the flutter speed is predicted on avergage about 10%too high. The two DLM based methods (labelled NAS-TRAN and ZONA6) produce very similar boundaries, although the NASTRAN results are generally 2 or 3% lower (and hence nearer the experimental boundary) across all speeds. ZTAIC produces a boundary very closer to that of ZONA6 in the subsonic region, but is notably poorer in the supersonic. However, the minima of flutter speed in the dip produced is considerably nearer the experimental value than either of the other two codes.

The results for the more advanced non-linear codes are given in Fig. 1(b). Agreement between RANSMB and PMB is good, as would be expected given the similarity of the methods, although RANSMB consistently underpredicts PMB. The correlation of the flutter bound-

	Case 1	Case 2	Case 3	Case 4
Mach Number	0.85	0.88	0.88	0.3
Lift Coefficient	0.4581	0.3263	0.1686	0.4581
Aircraft Mass	371  tons	537  tons	537  tons	-
Altitude	$11.27~\mathrm{km}$	$7 \mathrm{km}$	$2 \mathrm{km}$	$11.27~\mathrm{km}$

Table 1: Conditions for MDO test case. The lift coefficient quoted here is scaled in terms of the wing surface area,  $725m^2$ 

ary with experiment is greater than that demonstrated by the linear methods, even in the low speed regime, and particularly so in the transonic dip. Although significant errors (compared to experiment) are present in supersonic flow, this is consistent with other workers using similar methods [29,31].

#### The MDO Wing

The AGARD wing is symmetric and extremely thin, design features which increase the applicability of the linearised methods discussed, but reduce similarity with practical designs for transonic aircraft. In order to gauge the performance of the techniques discussed on a more realisitic design case, a second case was examined, this being the MDO wing. This presents a serious challenge to the linear methods, as the neglected aerofoil thickness plays a crucial role in aeroelastic behaviour.

The Multi-Disciplinary Optimisation (MDO) wing was extracted from the MDO aircraft designed in a Brite-Euram project to establish design methodology for future large commercial aircraft. It was used in the Unsteady Flow in the Context of Fluid-Structure Interaction (UNSI) project as a test case for coupled CFD-structural dynamics simulations. Comparison of the results between the different parters can be found in various publications of the UNSI partners [30–32]. The MDO configuration was optimised for high performance at a particular cruise condition. The optimising calculations were started from an estimate of the jig shape which leads to the design shape at cruise conditions. The wing has a semi-span of 36m, a thick supercritical section, and a planform as shown in figure 2.

The UNSI partners involved in the MDO comparing coupled analysis methods for analysis of the MDO wing were Alenia, BAE SYSTEMS, Dassault, ONERA, and Saab. A total of eight different solution methods were



Figure 2: MDO wing planform. The dark shaded region indicates the extent of the structural model.

used, including coupled Euler - NASTRAN analysis, and Euler, viscous full potential, and transonic perturbation schemes coupled to modal analysis. Static and dynamic test cases were devised, and the results produced showed some notable differences even between similar methods, without identifying the specific cause [31, 32]. Possible explanations included the accuracy with which the CFD scheme calculates the loading, the effectiveness and consistency of the load transfer algorithm, the accuracy of the structural calculation, and the accuracy with which the displacement algorithm transforms the aerodynamic shape (reference 32, page 318). The interpolation methods used were IPS (Infinite Plate Spline) or polynomial methods for ONERA, interpolation on an intermediate grid for Dassault and the use of rigid elements in the NASTRAN solver by Alenia.

Three cases were used for the comparisons and are summarised in table 1. An extra test case (4) was devised specifically for the current work at a Mach Number of



Figure 3: Leading and Trailing Edge Deflections, MDO Wing, Case 1

0.3 to allow comparison with linear methods employed away from the non-linear transonic region. In all respects except Mach number (and hence velocity) conditions were the same as in case 1. For each code the angle of attack is chosen to match the design lift coefficient. For cases 2 and 3, dynamic calculations were performed by both the UNSI partners and using the non-linear methods used in the current work, but are not included here as they principally provide a comparison between non-linear methods, rather than between linear and non-linear, the subject of this paper. They were also more difficult to interpret and quantify. Instead, unique to this investigaiton a calculation of the flutter boundary of the wing was made.

Comparison of the current methods is made with Saab and ONERA codes, which are representative of the results from the UNSI comparisons, and involve methodologies most similar to that of the coupled codes described here [30].

The Saab results were generated using the EURANUS code which solves on multiblock structured grids using explicit time stepping and multigrid. Central and upwind differencing options are available. The mesh is moved via a sequence of pre-determined perturbation grids (in this case based on 18 modes). The transfer is achieved by projection onto a neutral surface and interpolation to obtain the displacement of projected points. Pseudo-time stepping is used for unsteady calculations and pseudo iterations are used to remove sequencing effects between the fluid and structural solutions.

ONERA's CANARI code is a structured cell-centred

Euler solver, and was coupled to a NASTRAN derived 20 mode structural model to define structural motion for the transpiration based aerodynamic-structure interface. The ONERA aerodynamic grid contained 365541 nodes, and the SAAB grid 106425.

#### Selected Results

A subset of a large database of results are presented herein; the conclusions are based on the complete set. In all cases, the wing was structurally modelled as a box using finite elements, and the first 18 of the resulting mode shapes used in the calculations. The static results are produced through simulation of the artificially damped wing under the conditions prescribed, the flutter boundaries calculated by removing the damping and disturbing the wing from the steady state.

Although flutter calculations are the primary focus of this paper, a brief examination of the static cases is presented. This is done primarily because of the aforementioned discrepancy between other non-linear calculations, in an attempt to determine which of the four factors identified is responsible. This question is itself of importance, for as advances in materials and structures make more flexible aircraft possible, the need for accurate determination of pressure around deformed wings in cruise becomes greater. Results derived from a ZTAIC analysis are also included, as a comparison of the performance of the linear and non-linear methods is necessary if appropriate selection of design tool for this task is to be made.

Typical of the static results were those produced un-



Figure 4: Pressure Distributions, MDO Wing, Case 1

der Case 1 conditions. This required a lift coefficient of 0.4581 under conditions as specified in table 1, achieved with an incidence of 1.148 degrees using PMB, and 0.931 degrees with RANSMB. This compares to 0.745 and 0.8925 degrees for the same case generated by Saab and ONERA codes. The linear method (ZAERO), however, requires an incidence of 5.3 degrees. The leading and trailing edge displacements produced are shown in Fig. 3. Excellent agreement between RANSMB and PMB is demonstrated, with good agreement with the other two coupled codes. The ZAERO displacements also show good correlation, despite the much higher incidence. This is because although the incidence for a given lift over predicted, the test cases require a force balance, and hence total force required is similar for all methods, producing roughly equivalent deflections.

Figure 4 shows the pressure distributions at 20, 60,

and 90 percent of the span produced by the non-linear methods. Generally, agreement between the RANSMB and PMB solutions is notably better than that between Saab and ONERA, and lie between these other two. This includes the 90% span section, where deflections are of course greatest. This may be due to the better agreement in deflection in this region, as well as more accurate interpolation of aerodynamic grid. Near the root region, however, the forebody pressures produced by RANSMB match ONERA results more closely, whilst those of PMB are more similar to Saab. This implies that these differences are largely due to the differencing formulation (central differencing vs. upwinding), although it should be noted that the ON-ERA results were based on a finer grid than the other three (containing around 300,000 points as opposed to about 100,000). The discrepancy in pressure here between PMB and RANSMB is likely to be a part of the



Figure 5: Leading and Trailing Edge Deflections, MDO Wing, Case 4



Figure 6: The 80% Span section, MDO Wing, Case 4

reason for the difference in incidence needed to achieve the required lift coefficient. However, it is also notable that the shock wave position in the RANSMB solution is always slightly rearwards of that produced by PMB, and hence also results in a slightly greater lift for a given incidence. Results from Cases' 2 and 3 produced similar trends. Examination of aerofoil shape preservation demonstrated that the non-linear codes achieved a high degree of similarity.

Case 4 differs from the other cases in that it has not previously been examined by any other workers. Its primary function is to allow comparison of linear and non-linear methods within the subsonic regime, where linear methods would be expected to produce accurate results. The conditions were maintained identical to case 1, except that the Mach number (and hence velocity) was lowered to 0.3 (88.5 m/s).

The cell-centred method (RANSMB) again predicts a higher suction peak at a given incidence, although this feature is more noticable for this subsonic case. This means that the required lift coefficient of 0.4581 is achieved at only 1.60 degrees, somewhat lower than the 2.37 degrees required for PMB, and considerably lower than the 4.21 degrees predicted by ZAERO. Despite this, the coupled codes maintain very good agreement in terms of deflection, pressure distribution, and aerofoil section (see for example Figs. 5 and 6). The deflection predicted by the linear ZTAIC method is again similar.

The difference between upper and lower surface pressure coefficients is shown for the 90 % span section in



Figure 7: Pressure Differences, MDO Wing, Case 1 and 4, 90% Span

Fig. 7 for case 1 and 4. The tendency for the linear method to under predict the lift generated on the rear of the aerofoil is increased in the transonic regime. This explains the more similar incidence required for case 4 (only about twice the non-linear codes, as oppposed to over five for case 1).

<u>Summary of Trends</u> The results presented here and elsewhere allow the following conclusions to be drawn:

- Agreement between PMB and RANSMB in terms of pressure distributions and deformations is generally excellent. This in turn implies that flow solver methodology (upwind vs. cell centred) has less effect on coupled calculations than the interpolation and grid motion methods. The exception to this is the forebody pressures near the root, where solver methodology (cell centred vs. upwind) is the principle factor, cell-centred codes producing a higher suction. This is likely to be related to the relatively coarse grid used for these calculations (only 81x26 points on the surface of a fairly comples wing geometry).
- The linear method (ZTAIC) tends to require much higher incidences to generate the required lift. Examination of the pressure distributions reveals that this is due in part to a failure to predict the lift generated over the rear of the aerofoil, leading to an increased incidence, and thus a higher suction peak at the front. This effect is intensified in the transonic region due to the rooftop design, and presence of shock waves. The incorrect chordwise lift distribution also induces a greater twist along

the span. The deflections caused by the loading, however, are similar those produced by the nonlinear methods, as the total forces acting on the structural models is the same.

## Flutter Boundary

The flutter boundaries of the MDO wing predicted by the various linear and non-linear codes under investigation are shown in Fig. 8. The flutter speed assumes an atmospheric density of 1.006  $Kgm^{-3}$ , and calculations were carried out at zero incidence.

Agreement between PMB and RANSMB is generally very good. As was the case with the AGARD445.6 wing, RANSMB consistently predicts a lower flutter speed than PMB, but the difference is small over the majority of the Mach number range (about 1%). It is probable that this is caused by the previously noted tendancy of RANSMB to predict higher suction at the leading edge for a given angle of incidence. Under small oscillations about a nominal angle of attack, this will tend to destabalise the motion more rapidly, and hence produce a lower flutter boundary. This would also explain the greater discrepancy at the lowest Mach number (M = 0.3), as it was noted previously that this effect was greatest there.

Whilst no experimental data exists for comparison, the formulation of the methods, similarity of the RANSMB and PMB results, and AGARD445.6 wing results strongly suggest that the true behaviour of the MDO wing would be similar to that predicted by the nonlinear methods, at least up to the 'floor' of the dip



Figure 8: Flutter Boundaries,  $\alpha = 0^0, \rho = 1.006 kgm^-3$ 

(Mach 0.9). It is likely that with this case, as for the AGARD445.6, the supersonic flutter speed is considerably over predicted.

The flutter boundary predicted by NASTRAN'S DLM and ZAERO ZONA6 are again generally very similar (up to M = 0.95), as might be expected from the similarity of their formulation. However, the agreement of these methods with the more sophisticated non-linear codes is less impressive than that achieved with the AGARD445.6 wing, the 'transonic dip' being smeared to such an extent that it appears to effect even quite low Mach numbers (0.5 and greater). The minimum flutter speed predicted is very close to that of the nonlinear methods, but the dissimilarity of surrounding results suggest this may be coincidental.

The CFD influenced ZTAIC shows generally better agreement with the non-linear techniques, partcularly with respect to the narrowness of the dip, although this feature is still broader and at a higher Mach number than that predicted by the non-linear codes.

Whilst potentially more accurate, the run time required by the non-linear methods to predict the flutter boundary is considerably greater than the alternative techniques. It is also dependent upon a number of factors, which do not effect the linear methods. This is due to the time-marching nature of the former solutions, as opposed to the frequency domain analysis of the latter. These factors include time step and period of simulation (smaller time steps and longer periods increase accuracy but also increase run time), the allowable maximum velocity gap between a stable and unstable condition (determined by the nature of the flutter boundary crossing), and whether a rough location of the position of the flutter boundary is already known. The first two factors effect the length of time any given calculation takes, the second two the number of such calculations required to accurately locate the boundary. However, as a rough rule of thumb, for each point on the MDO flutter boundary NASTRAN calculations required about a minute, ZAERO about 30 minutes, and the non-linear techniques between 15 and 50 hours, depending upon the aforementioned factors.

#### Conclusions

The analysis of the AGARD 445.6 wing demonstrates that the linear methods may be used to achieve a reasonable estimation of the flutter boundary of a wing if it is of simple design. Even here, however, the coupled Euler-modal methods are demonstrated to produce results more similar to experiment.

Analysis of the results for the MDO wing reveals that PMB and RANSMB perform in a very similar manner, despite using different solver methodologies (upwind vs. cell centred). This in turn implies that the reason for the differing results found in previous studies are primarily due to the interpolation between the structural and aerodynamic grids. The only notable differences between RANSMB and PMB solutions appear to be caused by the relatively coarse nature of the grid near the leading and trailing edges.

The accuracy of the linear methods for this case is less certain, although the influence of CFD generated results on these techniques, as demonstrated by the ZTAIC flutter boundary, significantly reduces the difference between the linear and non-linear flutter boundary predictions. However, it should be noted that even with such methods, significant differences in pressure distributions (caused by the inverse design mehod of ZTAIC, required to maintain linearity) remain. The effect of these on design process and dynamic results (LCO's etc.) are likely to be significant.

#### Acknowledgements

This work has been carried out under the PUMA DARP's 2002 programme: financial support from EP-SRC, MoD, DTi, Airbus and BAE SYSTEMS is acknowledged. The authors are grateful to both academic and industrial members of the PUMA aeroelastics Focus Group for their assistance.

## REFERENCES

- K.J. Badcock G. Sim and B.E. Richards. Aeroelastic studies using transonic flow cfd modelling. International Forum on Aeroelasticity and Structural Dynamics 1995, pp 18.1-18.12, June 1995.
- [2] B.B. Prananta M.H.L. Hounjet and R.J. Zwaan. Thin layer navier-stokes solver and its application for aeroelastic analysis of an airfoil in transonic

flow. International Forum on Aeroelasticity and Structural Dynamics 1995, pp 1.1-1.9, June 1995.

- [3] B.B. Prananta and M.H.L. Hounjet. Aeroelastic simulation with advanced cfd methods in 2-d and 3-d transonic flow. Symposium Unsteady Aerodynamics 1996, RAeS, London, UK, 17-18 July 1996.
- [4] D.M. Schuster P.S. Beran and L.J. Huttsell. Application of the ens3dae euler/navier-stokes aeroelastic method. Technical Report No. 822, AGARD, March 1998. Numerical Unsteady Aerodynamic and Aeroelastic Simulation, pp. 3-1 - 3-11.
- [5] S. Oscher and S. Charkravarthy. Upwind schemes and boundary conditions with applications to euler equations in general geometries. *Journal of Computational Physics*, 50:447–481, 1983.
- [6] B. Van Leer. Towards the ultimate conservative difference scheme ii: Monotonicity and conservation combined in a second order scheme. *Journal* of Computational Physics, 14:361–374, 1974.
- [7] A. Jameson. Time dependent calculations using multigrid, with applications to unsteady flows past airfoils and wings. AIAA Paper 91-1596.
- [8] W. Schmidt A. Jameson and E. Turkel. Numerical solutions of the euler equations by finite volume methods using runge-kutta time stepping schemes. AIAA-Paper No. 81-1259, 1981.
- [9] A.L. Gaitonde. A dual-time method for the solution of the unsteady euler equations. The Aeronautical Journal of the Royal Aeronautical Society, pages 283–291, October 1994.
- [10] N.D. Melson M.D. Sanetrik and H.L. Atkins. Time-accurate navier-stokes calculations with multigrid acceleration. Technical Report CP 3224, NASA, 1993.
- [11] M. Geradin and D. Rixen. Mechanical Vibrations. John Wiley and Sons Ltd, Baffins Lane, Chichester, West Sussex PO19 IUD, England, 2nd edition, 1997.
- [12] K.J. Bathe. Finite Element Procedures in Engineering Analysis. Prentice Hall, Englewood Cliffs, New Jersey, 1982.

- [13] G.S.L. Goura. Time Marching Analysis of Flutter using Computational Fluid Dynamics. PhD thesis, Glasgow University, Departnment of Aerospace Engineering, 2001.
- [14] D.P. Jones. Force transfer in aeroelastic calculations. Technical Report AE047, University of Bristol, December 2002.
- [15] W.J. Gordon and C.A. Hall. Construction of curvilinear co-ordinates systems and applications to mesh generation. *International Journal for Numerical Methods in Engineering*, 7:461–477, 1973.
- [16] E. Albano and W.P. Rodden. A doublet-lattice method for calculating lift distributions on oscillating surfaces in subsonic flows. *AIAA Journal*, 7:279–285, 1969.
- [17] J. P. Giesing T.P. Kalman and W.P.Rodden. Subsonic unsteady aerodynamics for general configurations; part i, vol i - direct application of the nonplanar doublet-lattice method. Technical Report AFFDL-TR-71-5, Part I, Vol I, Air Force Flight Dynamics Laboratory, 1971.
- [18] W.P. Rodden J.P. Giesing and T.P. Kalman. Refinements of the nonplanar aspects of the subsonic double-lattice lifting surface method. J. Aircraft, 9:69–73, 1972.
- [19] D.D. Liu Y.F. Kao and K.Y. Fung. An efficient method for computing unsteady transonic aerodynamics of swept wings with control surfaces. J. Aircraft, 52(1):25–31, 1988.
- [20] E.C. Yates Jr. Standard aeroelastic configurations for dynamic response i - wing 445.6. Technical Report 765, AGARD, Sept 1985. Includes Tech Report TN-D-1616 as an Appendix.
- [21] Version 3.2. ZAERO Applications Manual. Zona technology, arizona.
- [22] R.G.A da Silva and O.A. de Faria Mello. Prediction of transonic flutter using nastran with aerodynamic coefficients tuned to navier stokes computations. CEAS/AIAA/ICASE/NASA Meeting on Aeroelasticity and Structural Dynamics. Norfolk,Va,USA, June 1999.
- [23] J.T. Batina R.M. Bennett D.A. Seidel H.J. Cunningham and S. Bland. Recent advances in transonic computational aeroelasticity. *Computers* and Fluids, 30(172):29–37, 1998.

- [24] J.T. Edwards. Calculated viscous and scale effects on transonic aeroelasticity. presented at AGARD SMP meeting on Numerical Unsteady Aerodynamic and Aeroelastic Simulation held in Aalborg, Denmark, Published in AGARD R-822, October 1997.
- [25] K.K. Gupta. Development of a finite element aeroelastic analysis capability. *Journal of Aircraft*, 33(5):995–1002, 1996.
- [26] R.B. Melville S.A Morton and D.P. Rizzetta. Implementation of fully-implicit aeroelastic navierstokes solver. 13th Computational Fluid Dynamics Conference, 29th June - 2 July, Snowmass Village, CO. AIAA paper 97-2039, 1997.
- [27] R.D. Rausch J.T. Batina H.T. Yang. Threedimensional time-marching aeroelastic analyses using an unstructured-grid euler method. AIAA Journal, 31(9):1626–1633, 1993.
- [28] M. Lesonne and C. Farhat. High order subiteration-free staggered algorithm for nonlinear transient aeroelastic problems. *AIAA Journal*, 36(9):1754–1757, 1998.
- [29] R.E. Gordnier and R.B. Melville. Transonic flutter simulations using an implicit aeroelastic solver. *Journal of Aircraft*, 37(5):872–879, 2000.
- [30] P. Girodoux-Lavigne J.P. Grisval S. Guillemot M. Henshaw A.Karlsson V. Selmin J. Smith E. Teupootahiti and B. Winzell. Comparative study of advanced fluid-structure interaction methods in the case of a highly flexible wing( results from the unsi program). International Forum aeroealsticity structural dynamics, Madrid, June 2001.
- [31] P. Girodoux-Lavigne J.P. Grisval S. Guillemot M. Henshaw A.Karlsson V. Selmin J. Smith E. Teupootahiti and B. Winzell. Comparison of static and dynamic fluid-structure interaction solutions in the case of a highly flexible modern transport aircraft wing. Journal of Aerospace Science and Technology, 7:121–133, 2003.
- [32] W. Haase V. Selmin and B. Winzell, editors. Progresss in Computational Flow-Structure Interaction, volume 81 of Notes on Numerical Fluid Mechanics and Multidisciplinary Design. Springer-Verlag, 2003.