

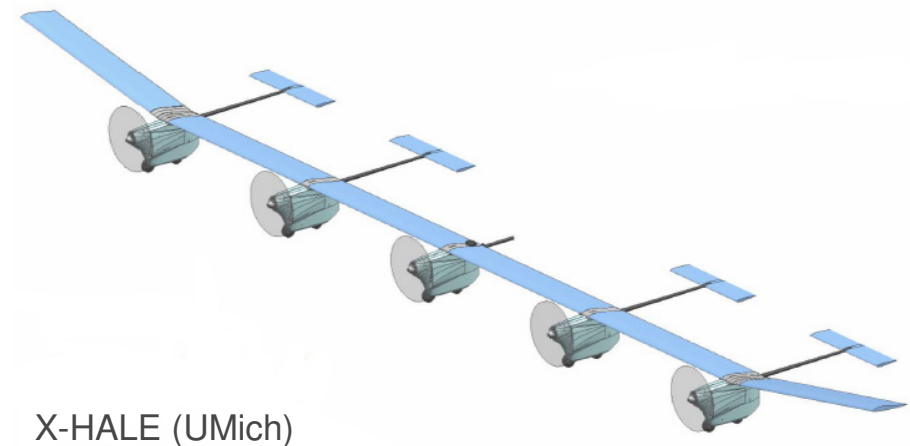
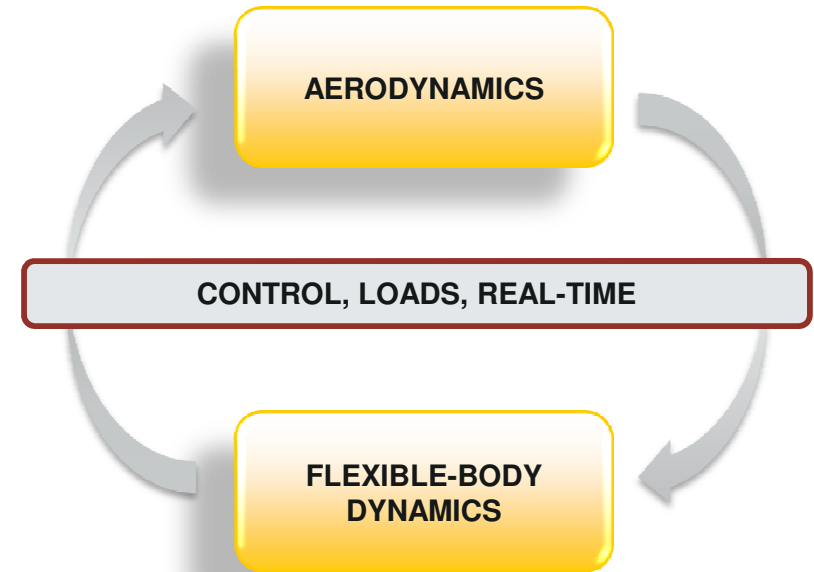
# **Model Reduction in Flexible-Aircraft Dynamics with Large Rigid-Body Motion**

**Henrik Hesse and Rafael Palacios**

Department of Aeronautics, Imperial College London

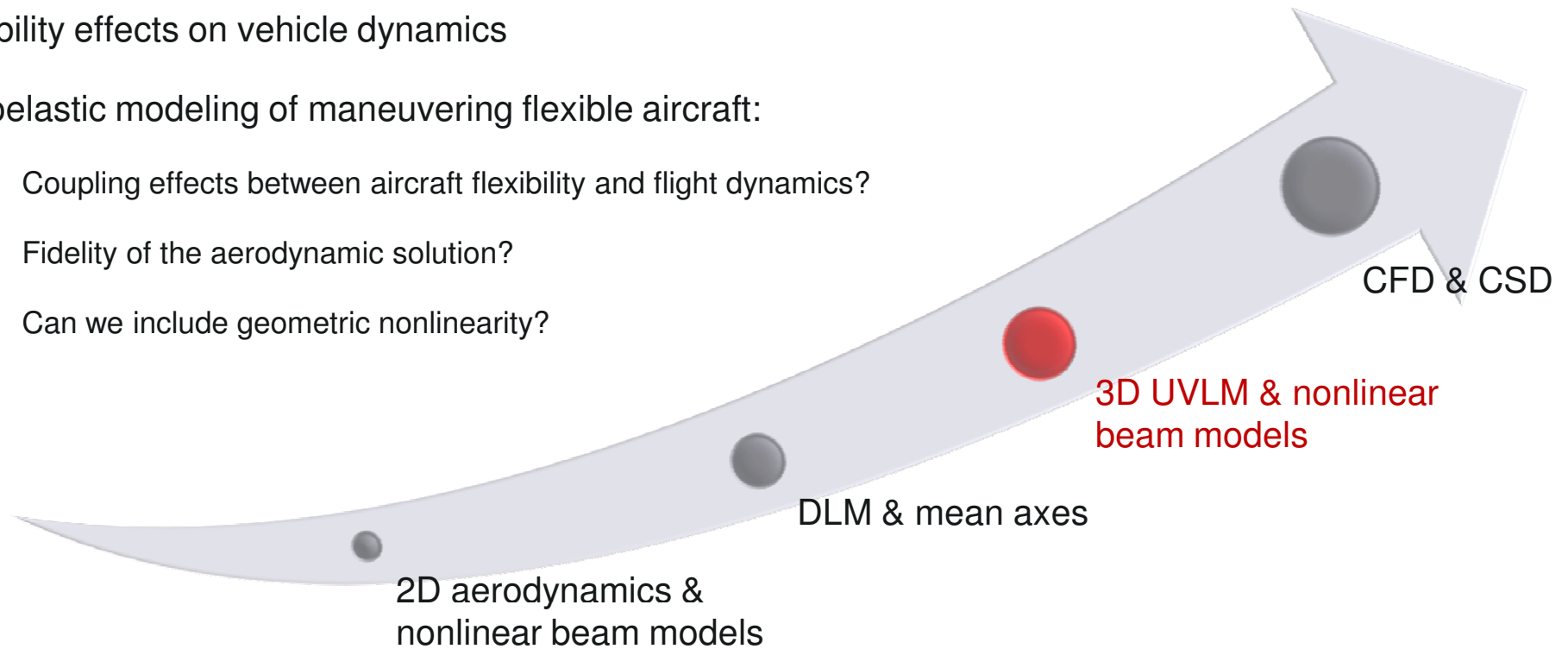
## Flexibility = Barrier to high efficiency

- Next-generation aircraft design requires incorporation of flexibility effects on vehicle dynamics
- Aeroelastic modeling of maneuvering flexible aircraft:
  - Coupling effects between aircraft flexibility and flight dynamics?
  - Fidelity of the aerodynamic solution?
  - Can we include geometric nonlinearity?



# Modeling of Flexible Aircraft Dynamics

- Next-generation aircraft design requires incorporation of flexibility effects on vehicle dynamics
- Aeroelastic modeling of maneuvering flexible aircraft:
  - Coupling effects between aircraft flexibility and flight dynamics?
  - Fidelity of the aerodynamic solution?
  - Can we include geometric nonlinearity?



- Time-domain methods provide answers, but are computationally expensive with large system sizes
  - Required model fidelity and model reduction for control synthesis and load calculations?

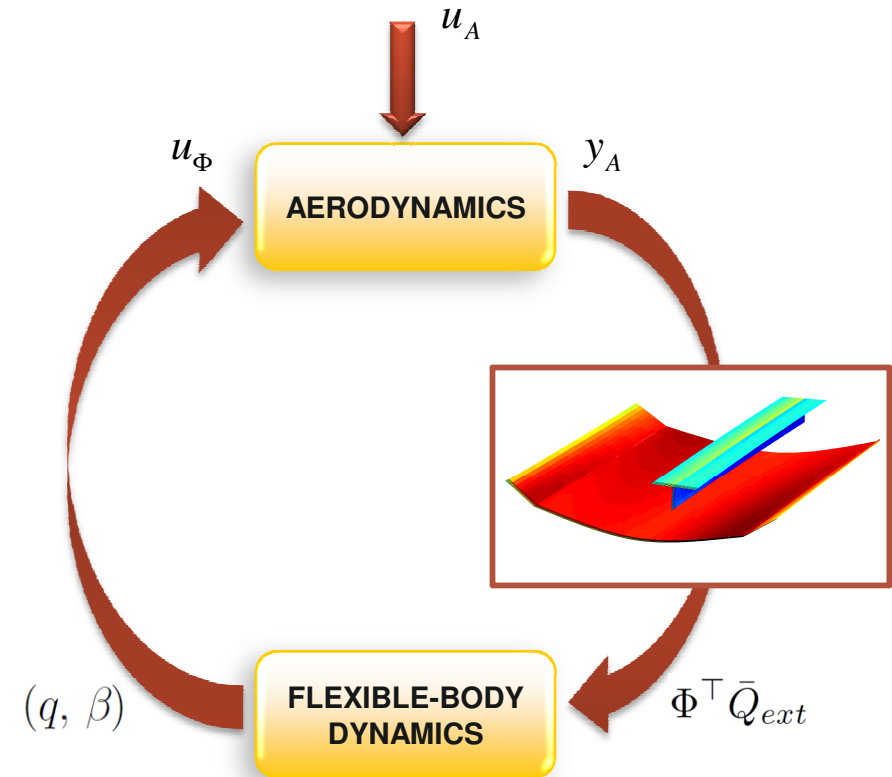
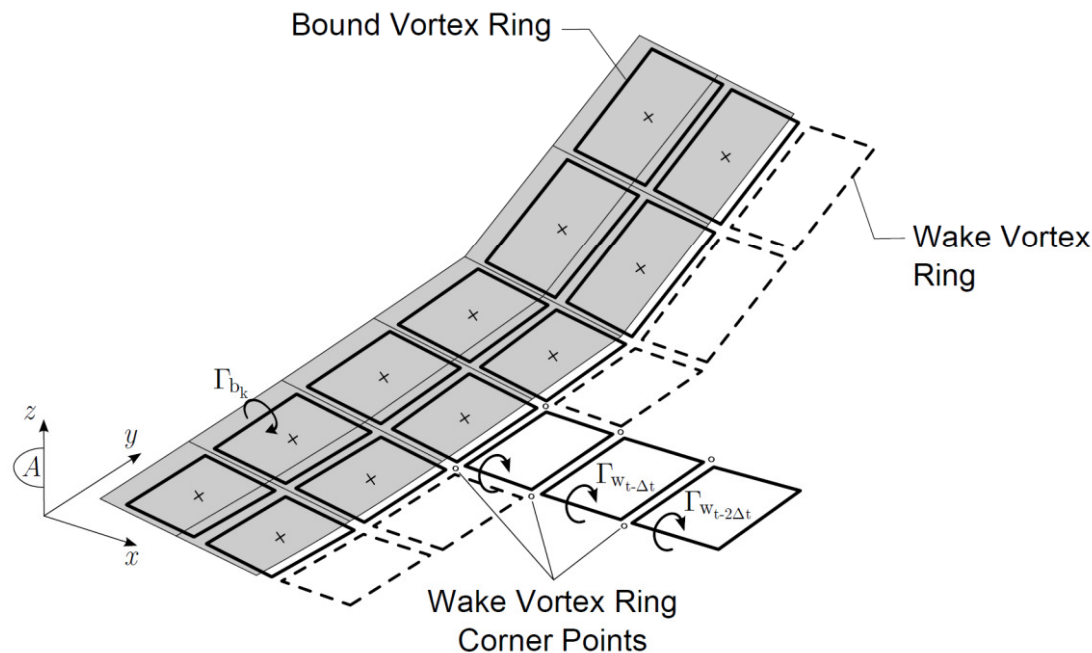
# Aeroelastic system for maneuvering aircraft

$$\Delta\Gamma^{n+1} = A\Delta\Gamma^n + B_S\Phi\Delta u_\Phi^n + B_A u_A^n$$

$$\Delta y_A^n = \Phi^T (C\Delta\Gamma^n + D_S\Phi\Delta u_\Phi^n + D_A u_A^n)$$

## Linearised UVLM<sup>1</sup>

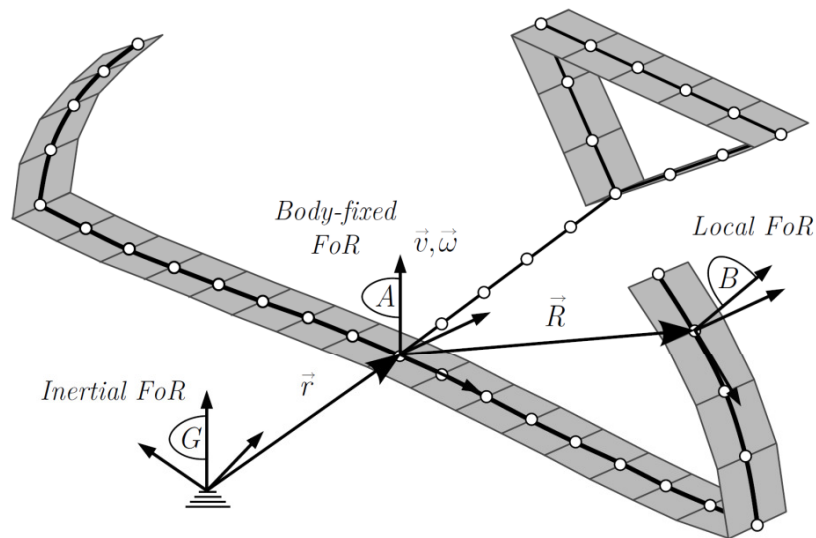
- Small deformations and velocities
- Low speed maneuvers



# Aeroelastic system for maneuvering aircraft

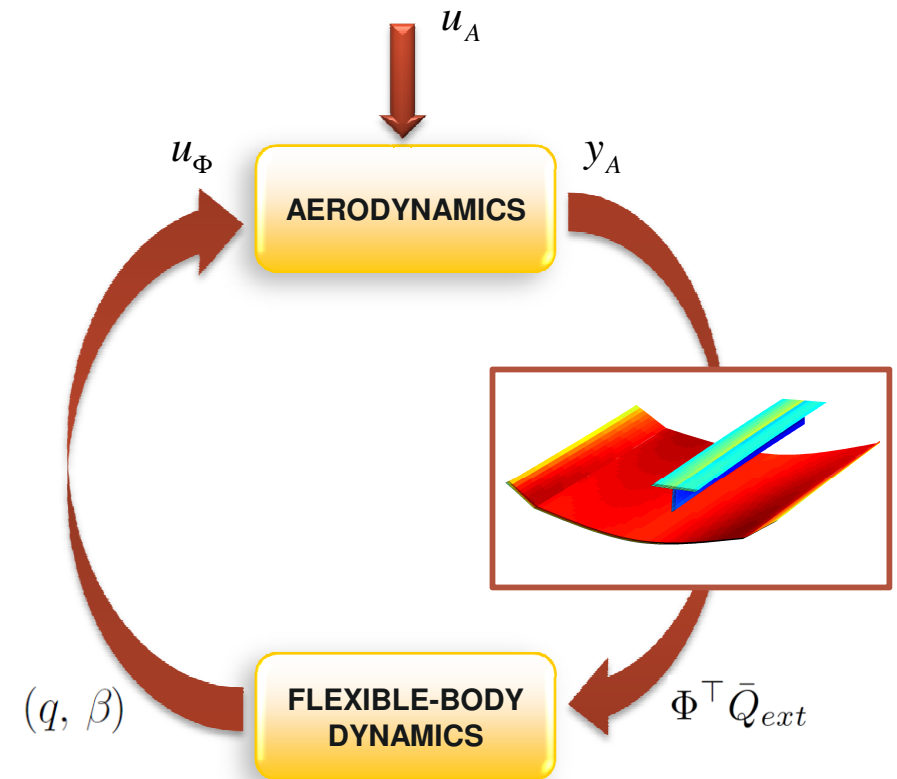
## Perturbation flexible-body dynamics

- Composite beam elements on a moving **body-attached** reference frame
- Linearisation of **structural** DoF around nonlinear trim configuration  $\eta_0$



### Equation of motion

$$M(\eta_0) \begin{Bmatrix} \ddot{\bar{\eta}} \\ \dot{\beta} \end{Bmatrix} + \bar{C}(\eta_0, \beta) \begin{Bmatrix} \dot{\bar{\eta}} \\ \beta \end{Bmatrix} + \bar{K}(\eta_0, \beta) \begin{Bmatrix} \bar{\eta} \\ 0 \end{Bmatrix} = \bar{Q}_{ext}(\bar{\eta}, \dot{\bar{\eta}}, \beta, \zeta)$$



### Rigid-body DoF

$$\beta = \begin{Bmatrix} v_A \\ \omega_A \end{Bmatrix}$$

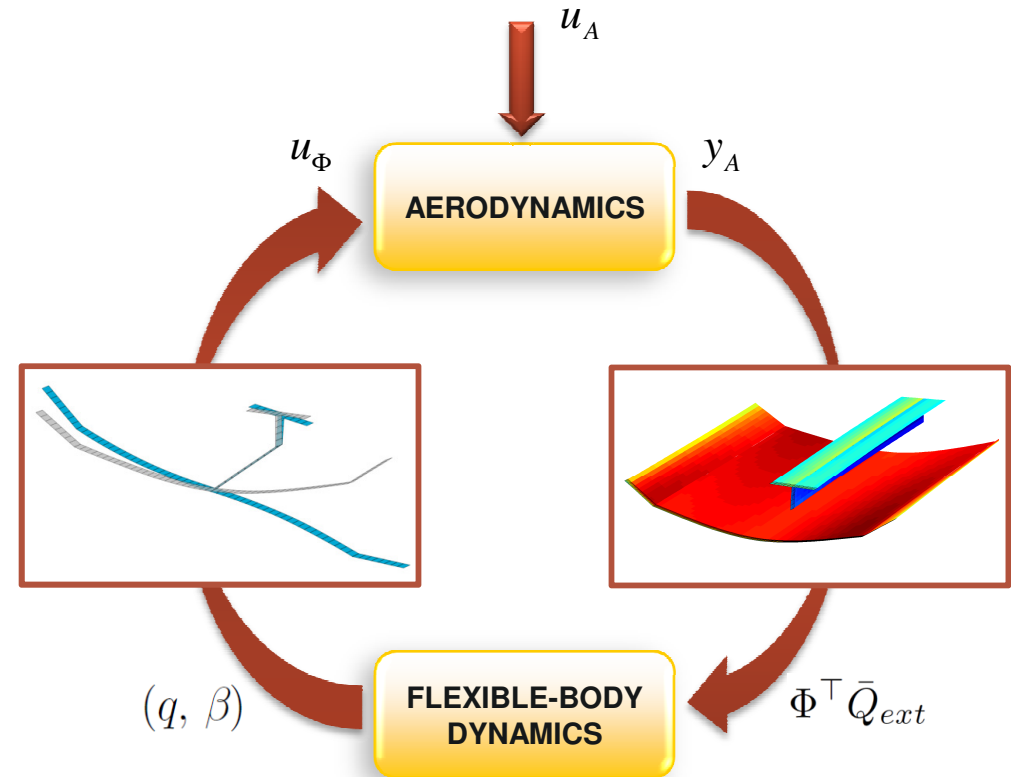
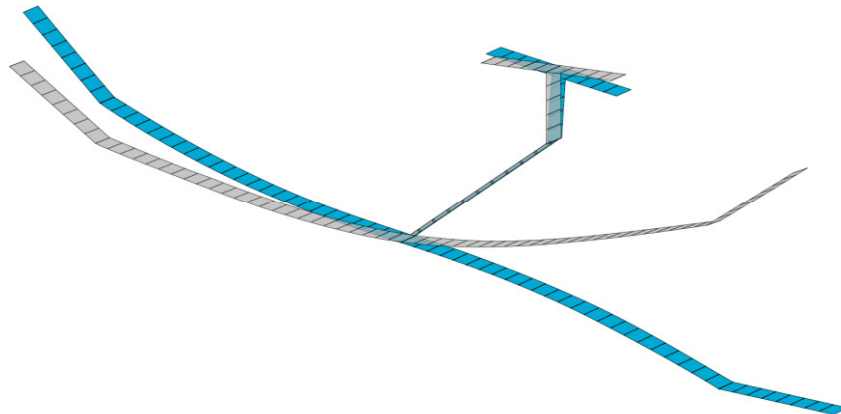
### Structural DoF

$$\bar{\eta} = \begin{Bmatrix} \bar{R}_A \\ \bar{\Psi} \end{Bmatrix}$$

# Aeroelastic system for maneuvering aircraft

## Perturbation flexible-body dynamics

- Composite beam elements on a moving **body-attached** reference frame
- Linearisation of **structural** DoF around nonlinear trim configuration  $\eta_0$
- Truncate number of generalised coordinates passed to the aerodynamic system



### Equation of motion in modal form

$$\Phi^T M(\eta_0) \Phi \begin{Bmatrix} \ddot{q} \\ \dot{\beta} \end{Bmatrix} + \Phi^T \bar{C}(\eta_0, \beta) \Phi \begin{Bmatrix} \dot{q} \\ \beta \end{Bmatrix} + \Phi^T \bar{K}(\eta_0, \beta) \Phi \begin{Bmatrix} q \\ 0 \end{Bmatrix} = \Phi^T \bar{Q}_{ext}(\bar{\eta}, \dot{\bar{\eta}}, \beta, \zeta)$$

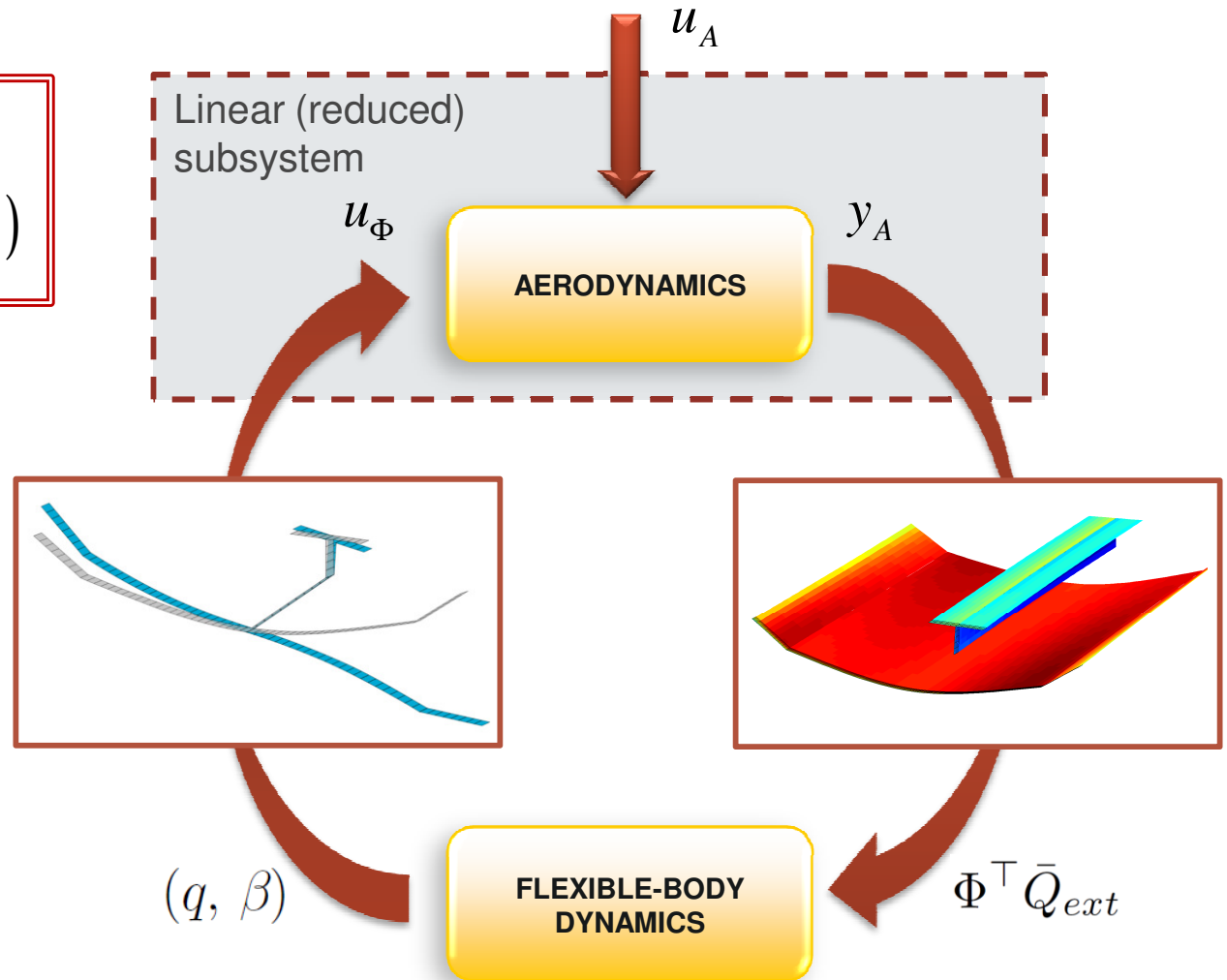
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## Linearised UVLM

- Small deformations and velocities
- Low speed maneuvers

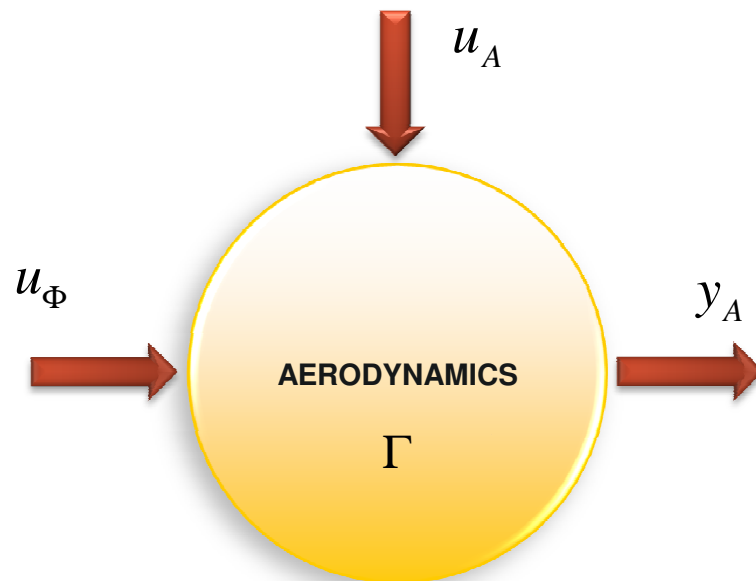


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## Balanced truncation of the aeroelastic system

- Large aerodynamic system ideal for balanced truncation
  - Few inputs and outputs transmitted by large system ( $10^4$ )



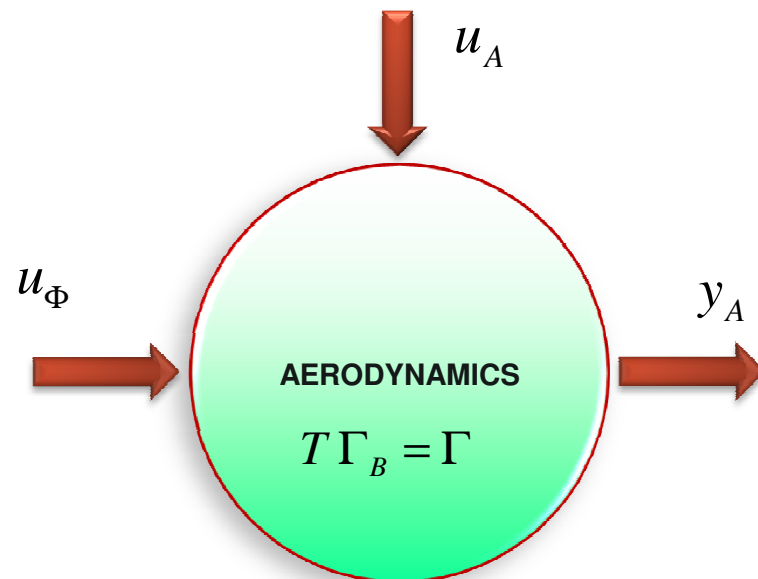
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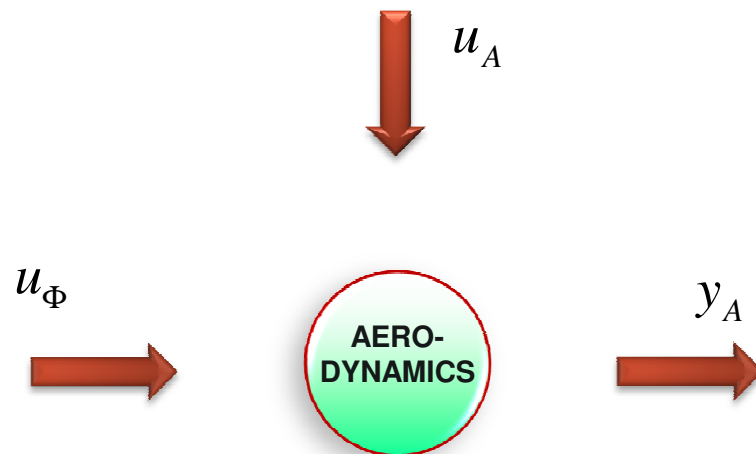


$$\Delta\Gamma_B^{n+1} = T^{-1}AT\Delta\Gamma_B^n + T^{-1}B_S\Phi\Delta u_\Phi^n + T^{-1}B_A u_A^n$$

$$\Delta y_A^n = \Phi^T \left( CT\Delta\Gamma_B^n + D_S\Phi\Delta u_\Phi^n + D_A u_A^n \right)$$

## Balanced truncation of the aeroelastic system

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  - **Truncate least controllable and observable states**



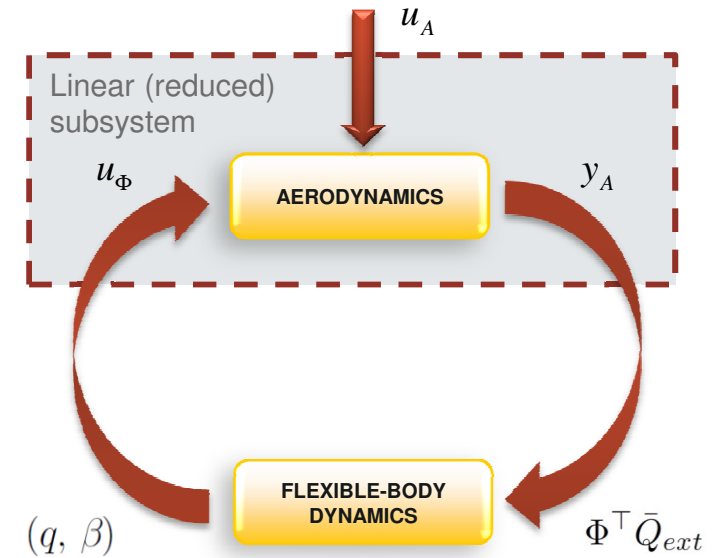
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# Aeroelastic system for aircraft analysis

## 1. Aeroelastic system for maneuvering flexible aircraft

- Suitable for **large** rigid-body angular velocities with resulting coupling terms in the elastic deformations
- Dynamic load calculations due to gust and **maneuver**, real-time simulations, optimisation, and control



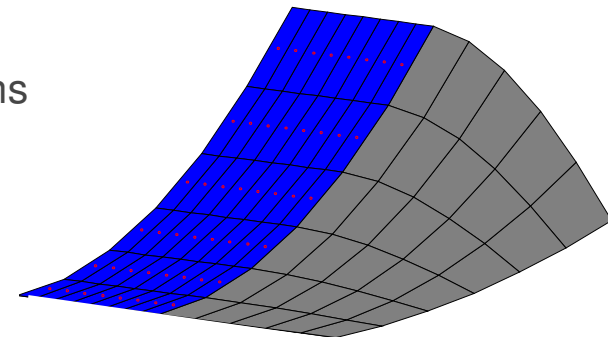
## 2. Monolithic framework of the integrated aeroelasticity and flight dynamics

- Control synthesis, stability analysis, and optimisation
- Suitable for **clamped** problems or **linear** rigid-body motions

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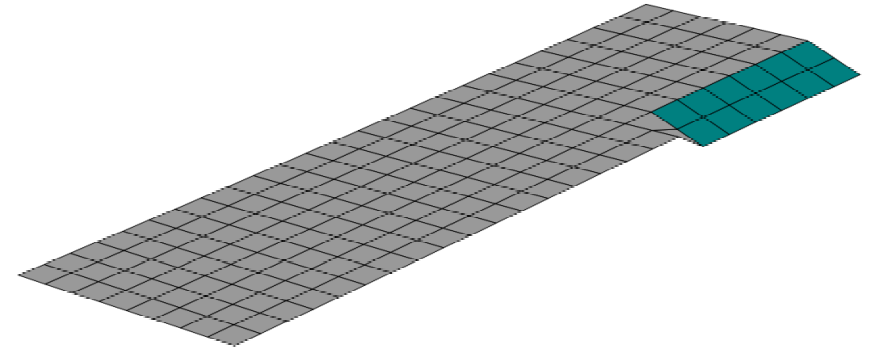
$$x = \left[ x_A^T \mid x_S^T \right]^T$$



# Numerical studies

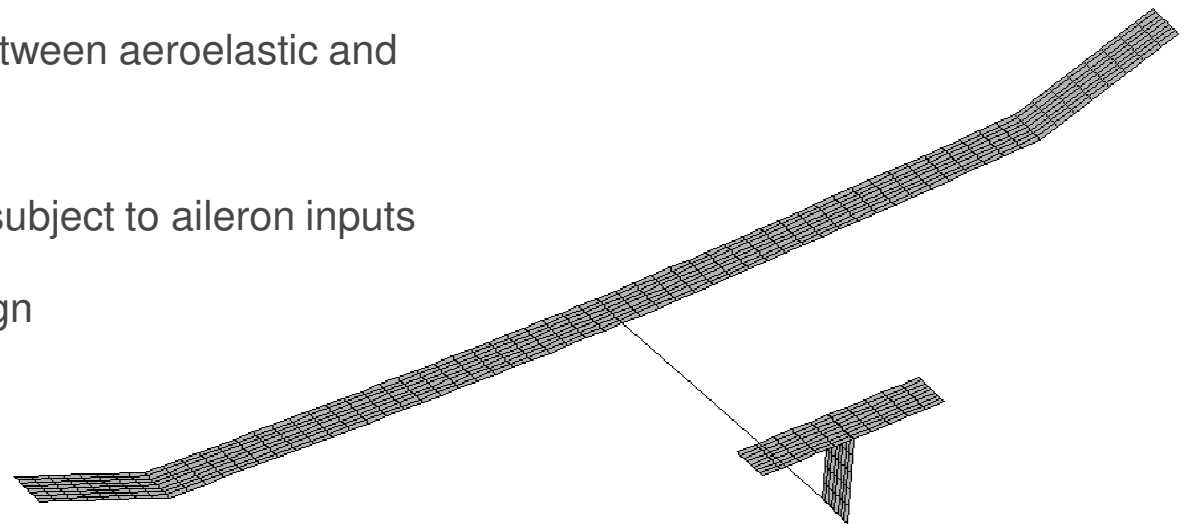
## 1. Goland wing with control surfaces

- Verification of the linearized aeroelastic approach
- Demonstration of balanced truncation on monolithic framework
- Robust control synthesis based on ROM



## 2. Representative HALE aircraft

- Insight into the coupling effects between aeroelastic and rigid-body modes
- ROM of the maneuvering aircraft subject to aileron inputs
- Generic approach for aircraft design

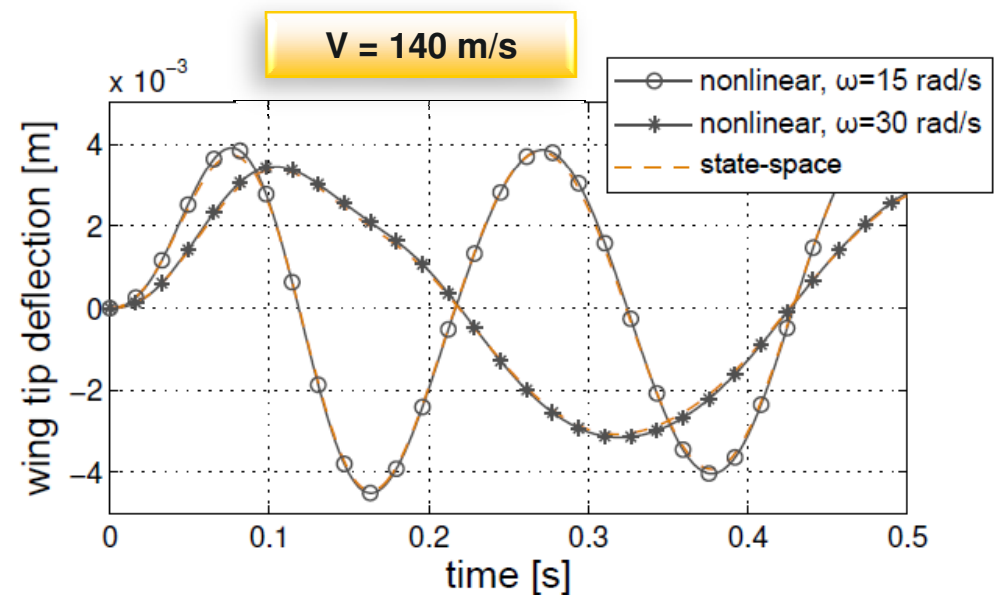
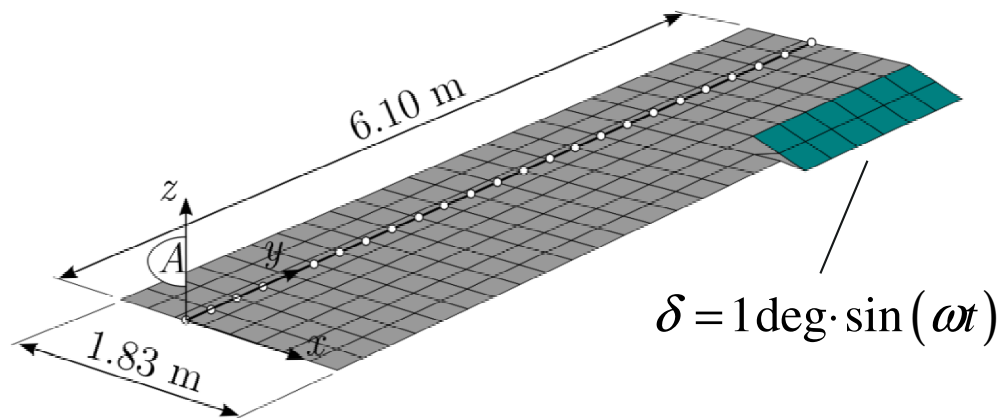


# Goland wing

- Benchmark for aeroelastic calculations (Goland, 1945)
- Relatively stiff and low aspect-ratio wing
- Flutter speed using 16 x 26 bound panels (4550 states)
  - Present approach:  $V_f = 169 \text{ m/s}$      $\omega_f = 70 \text{ rad/s}$
  - Murua et al. (2010):  $V_f = 165 \text{ m/s}$      $\omega_f = 72 \text{ rad/s}$
  - Wang et al. (2006):  $V_f = 164 \text{ m/s}$     -
- Comparison with nonlinear time-marching solution

## Goland wing characteristics

Aspect ratio	3.33
Elastic axis (from le)	33 %
Center of gravity (from le)	43 %
Mass per unit length	35.71 kg/m
Torsional rigidity	$0.99 \times 10^6 \text{ N}\cdot\text{m}^2$
Bending rigidity	$9.77 \times 10^6 \text{ N}\cdot\text{m}^2$

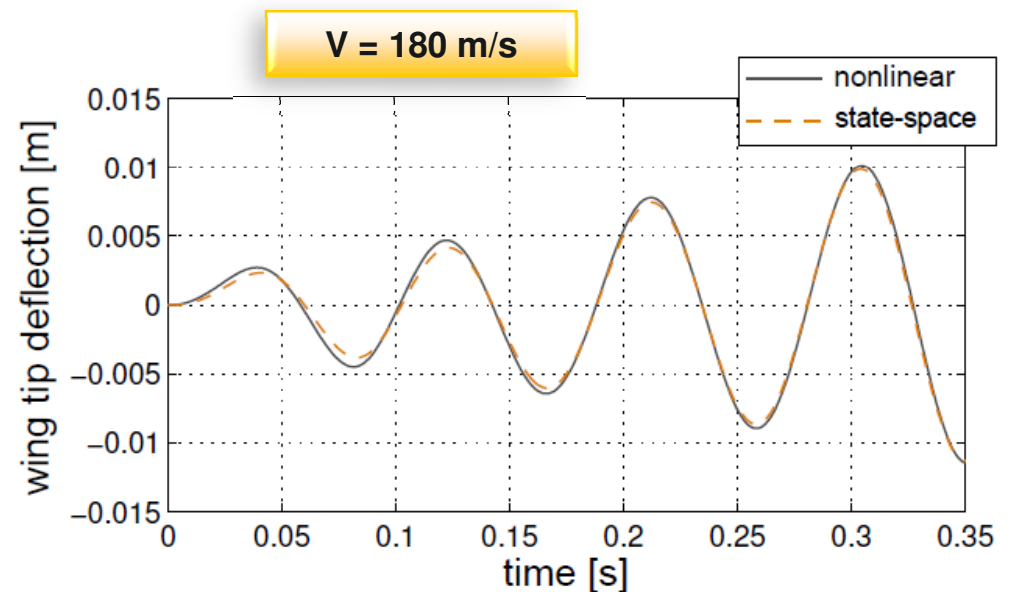
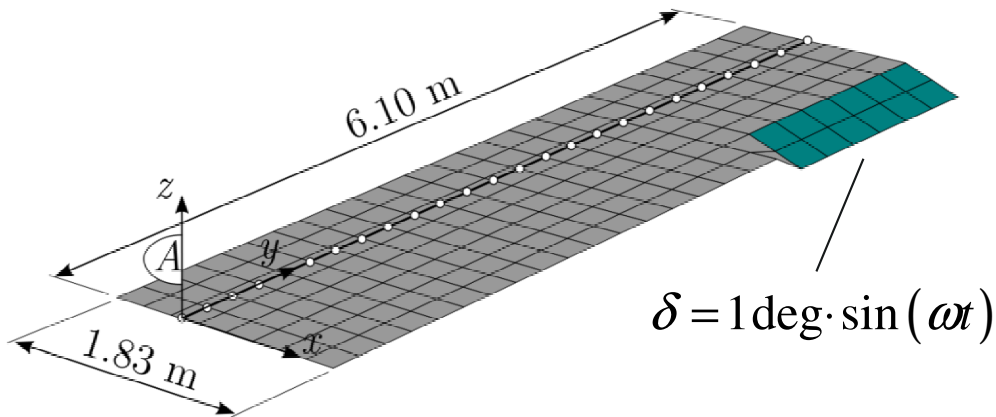


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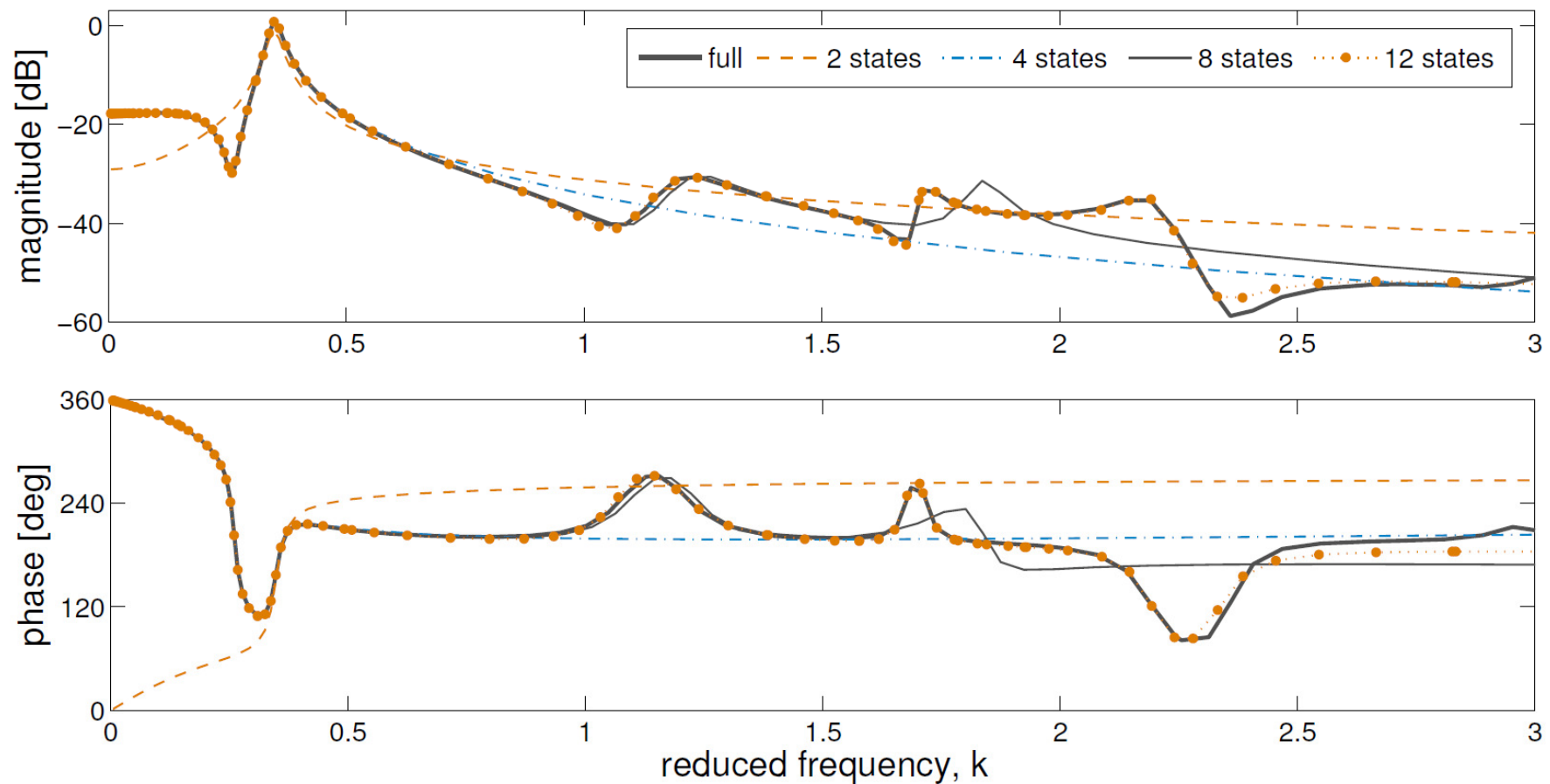
**State-space system predicts high frequency aeroelastic response**

**But: large system size obstacle for effective aircraft design**



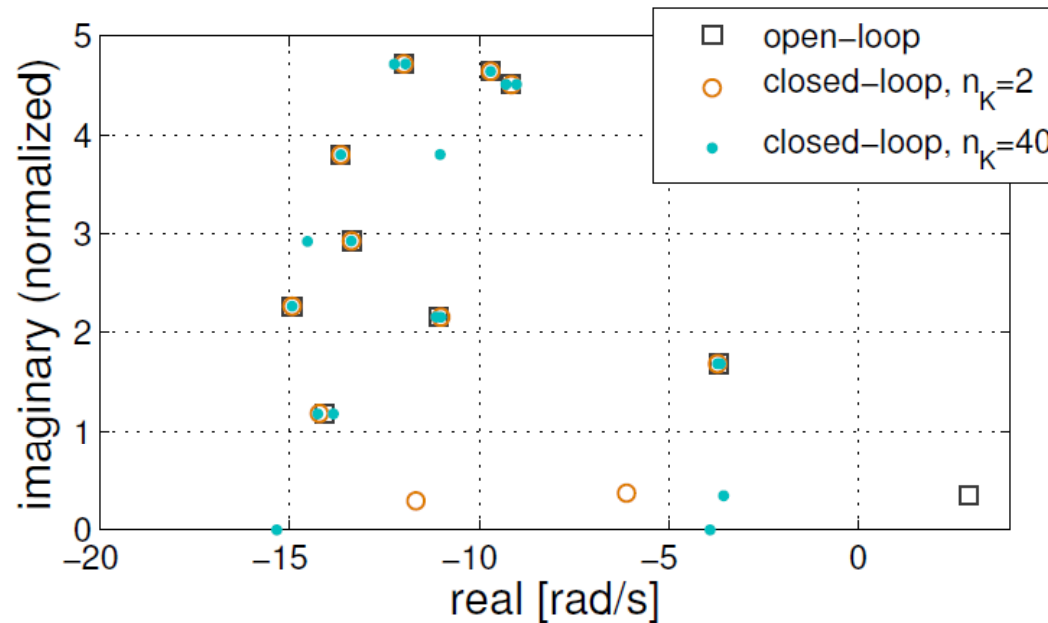
# Model reduction for the Goland wing

- Balanced truncation of the SISO aeroelastic system at  $V = 180$  m/s with 4550 states with aileron input and tip deflection as output



## Control synthesis for the Goland wing

- Robust control synthesis using  $H_\infty$  for suppression of structural vibrations
- Demonstrate flutter suppression of the Goland wing at  $V=180$  m/s



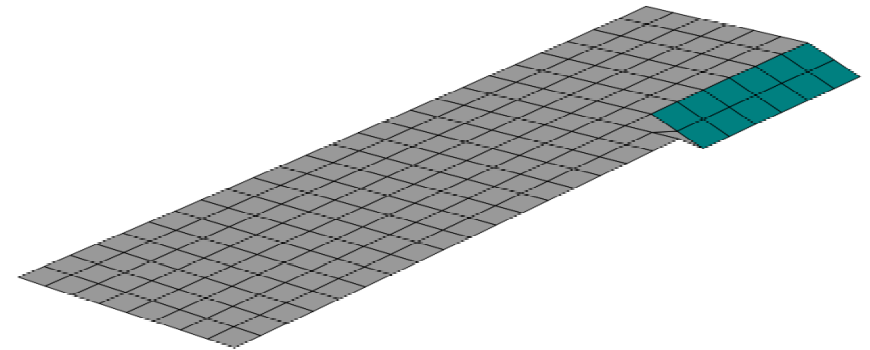
**Model reduction enables use of higher fidelity tools to reduce uncertainties**



# Numerical studies

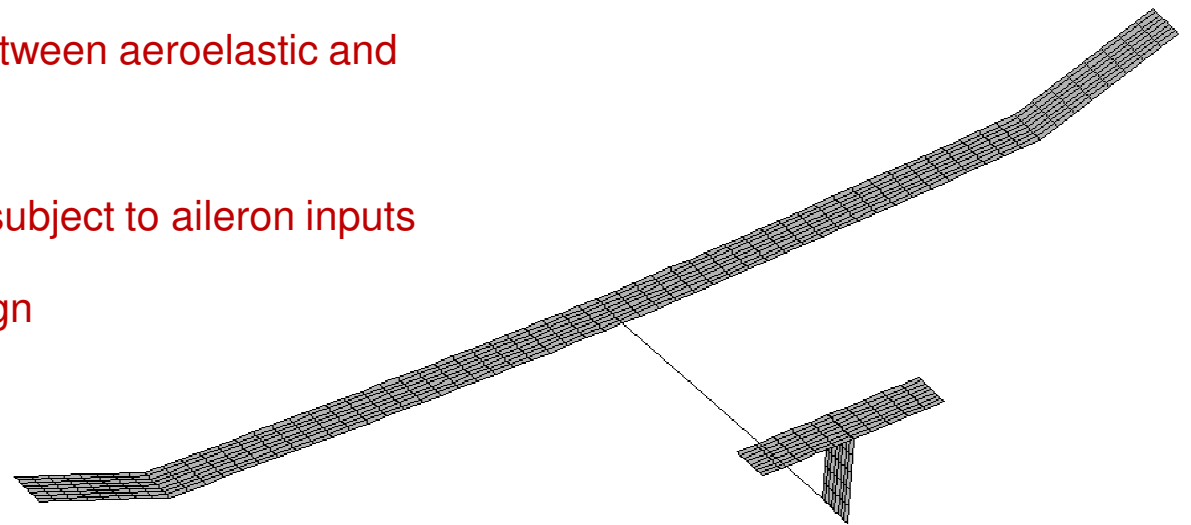
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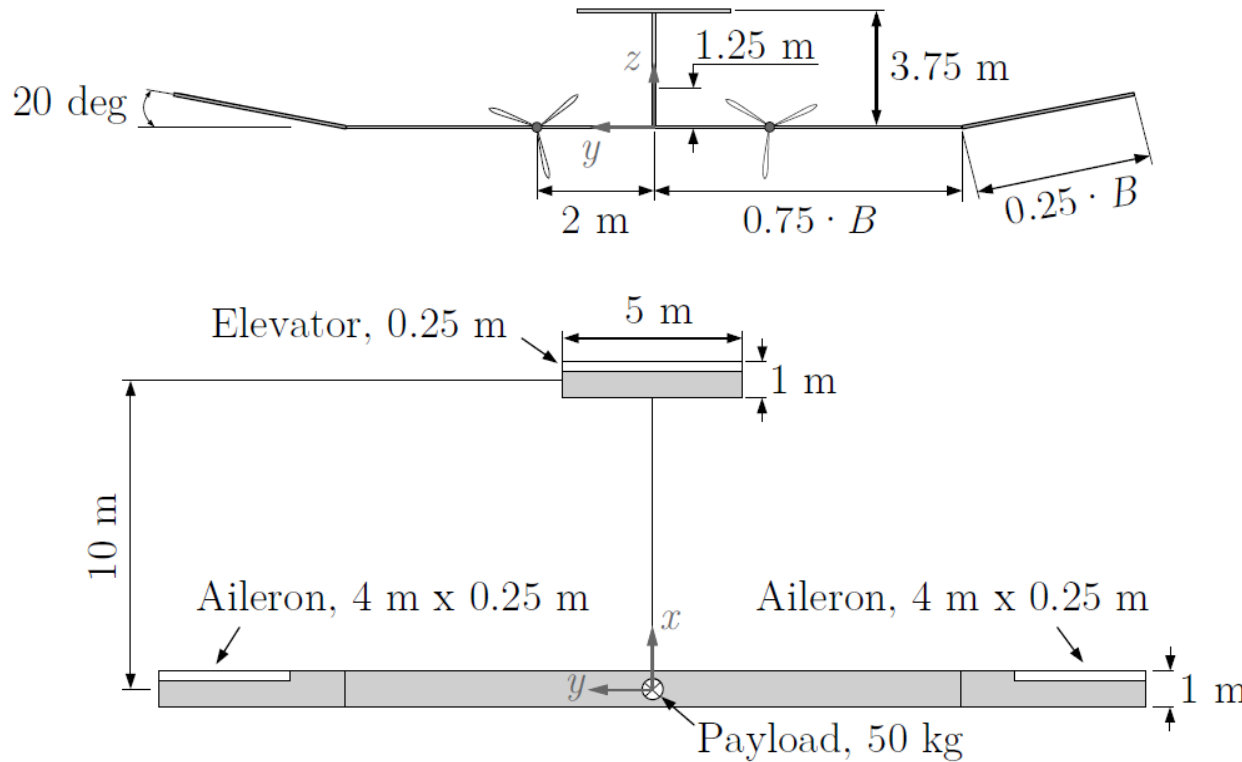


## 2. Representative HALE aircraft

- Insight into the coupling effects between aeroelastic and rigid-body modes
- ROM of the maneuvering aircraft subject to aileron inputs
- Generic approach for aircraft design



# Representative HALE UAV



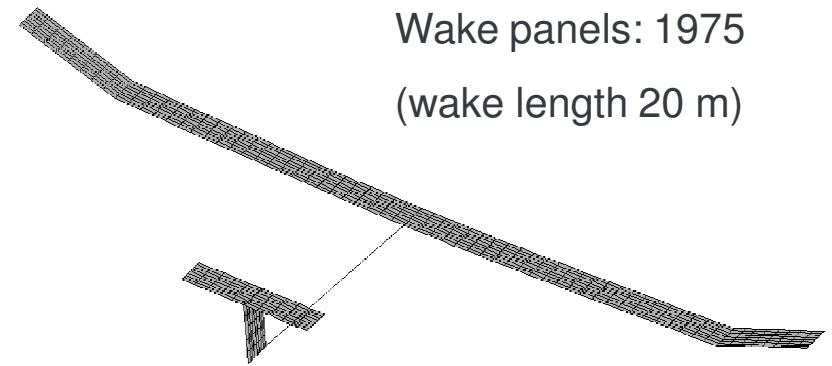
## HALE model characteristics

Aspect ratio	16
Elastic axis (from le)	50 %
Center of gravity (from le)	50 %
Mass per unit length	0.75 kg/m
Torsional rigidity	$1\sigma \times 10^4 \text{ N}\cdot\text{m}^2$
Bending rigidity	$2\sigma \times 10^4 \text{ N}\cdot\text{m}^2$

Bound panels: 632

Wake panels: 1975

(wake length 20 m)



Free-stream velocity: 30 m/s with time step 0.02 s

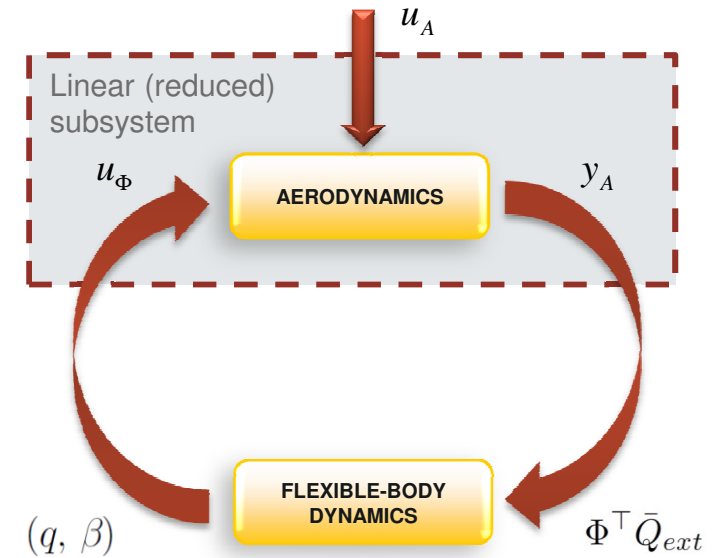
Flexible main wing

Rigid fuselage and T-tail

# Aeroelastic system for aircraft analysis

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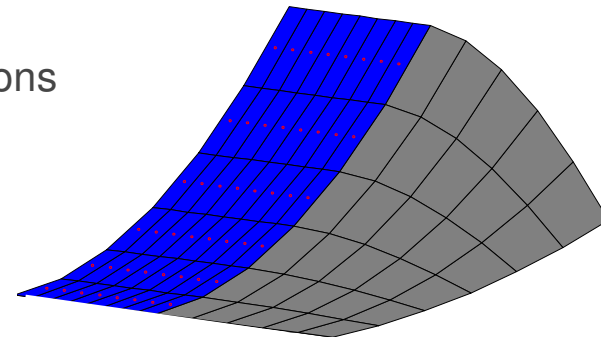
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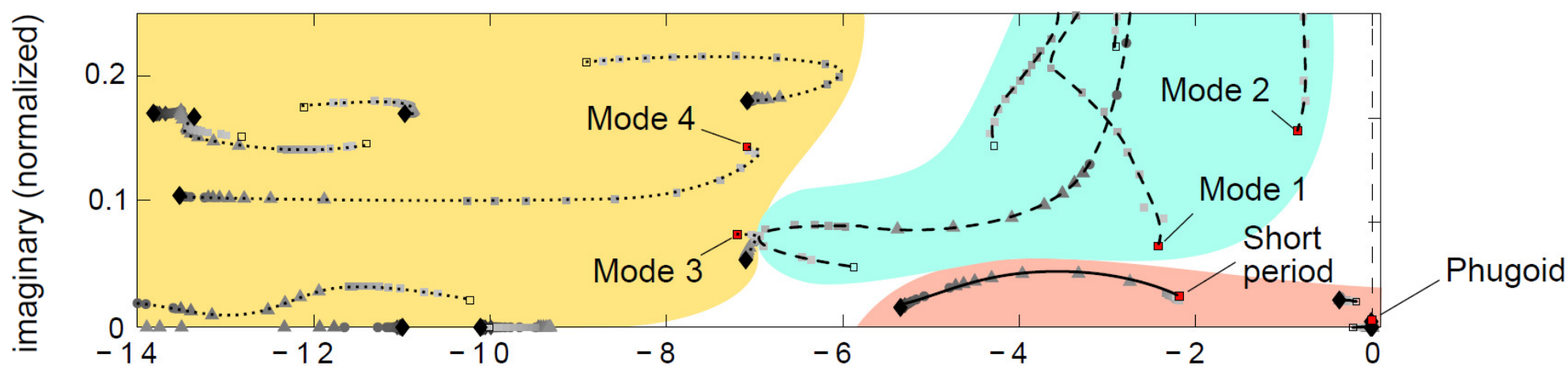
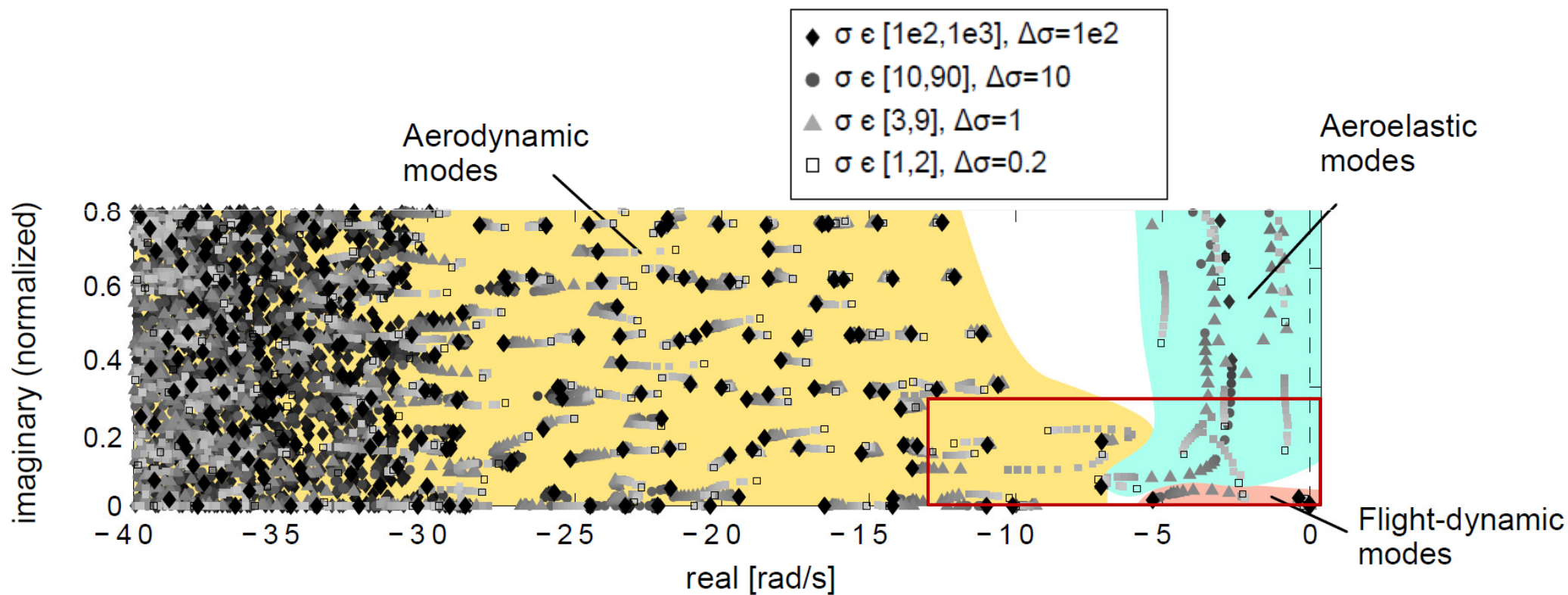
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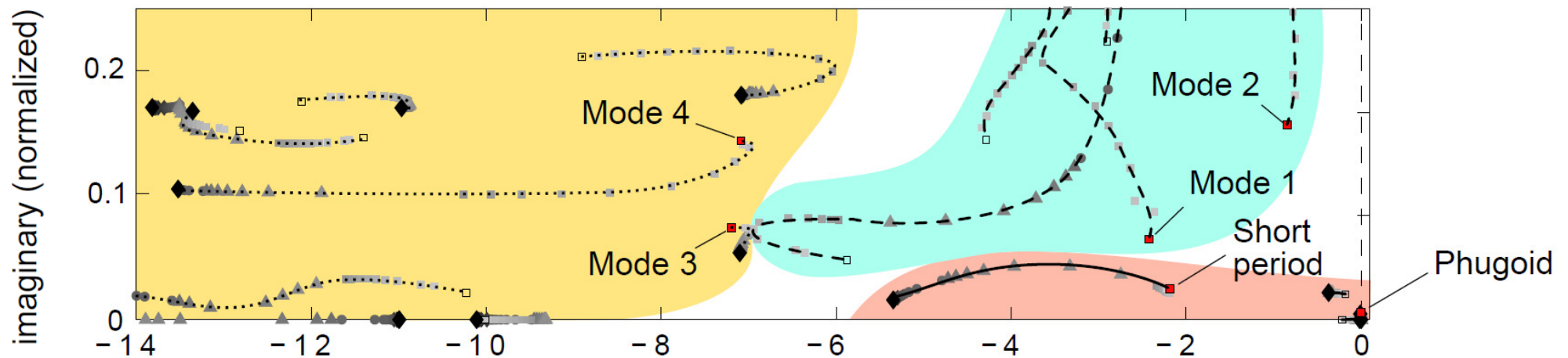
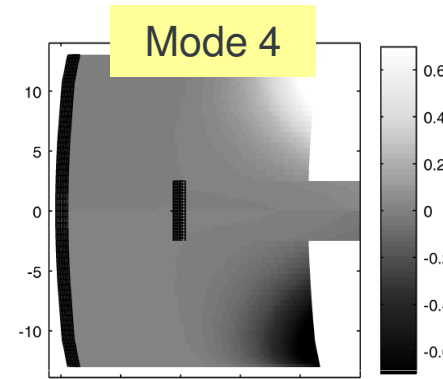
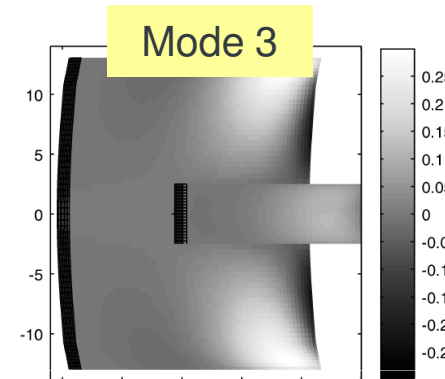
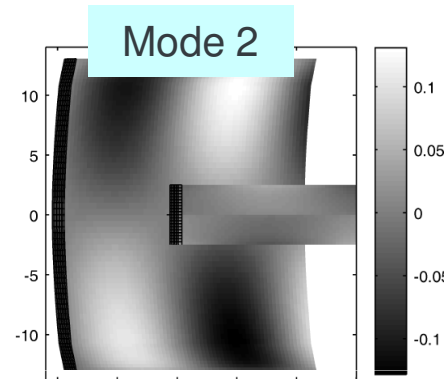
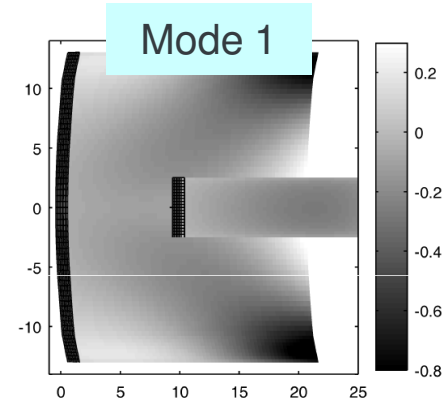
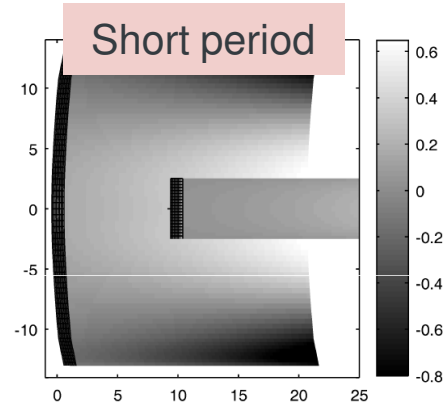
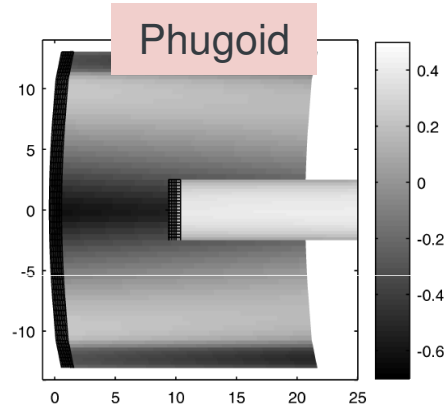
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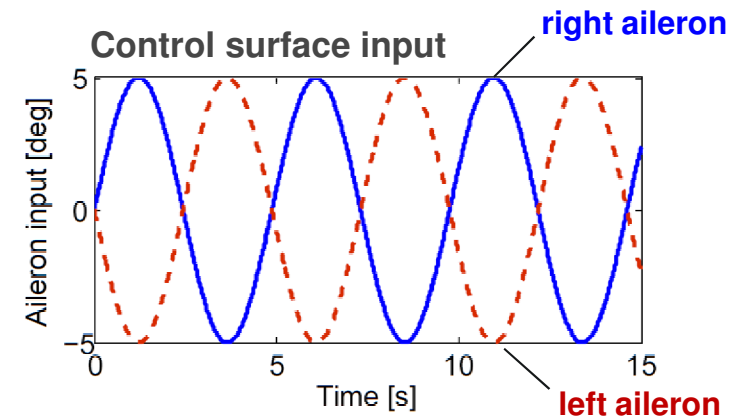
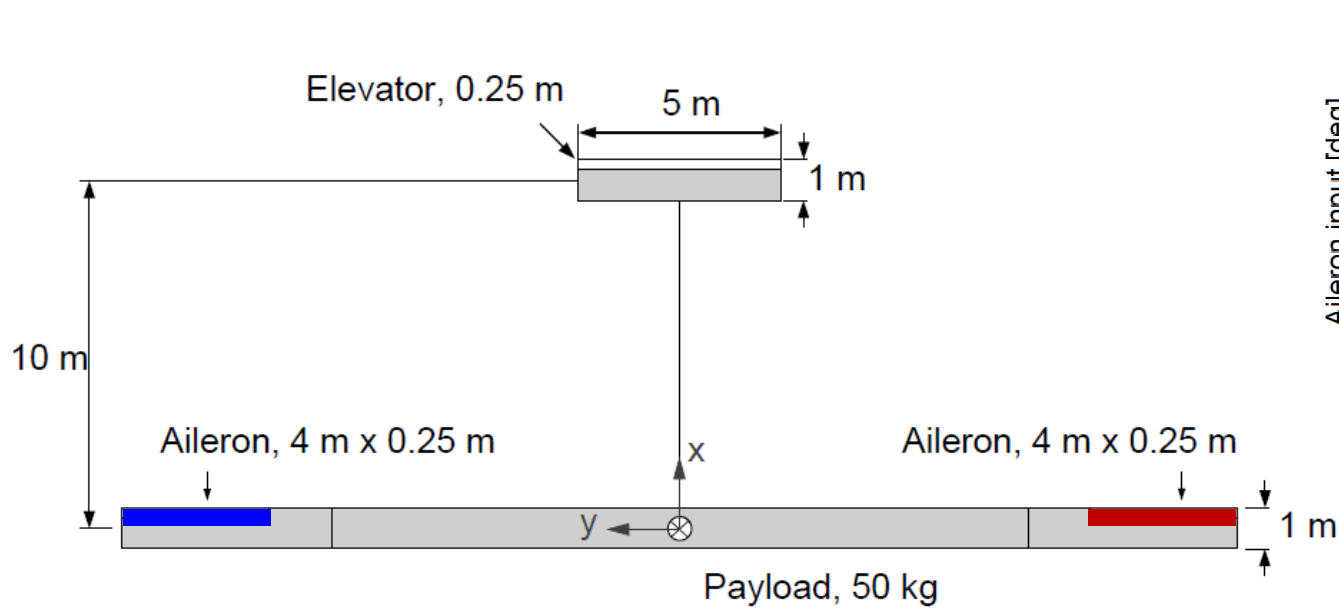
# Coupled aeroelastic/flight dynamic stability



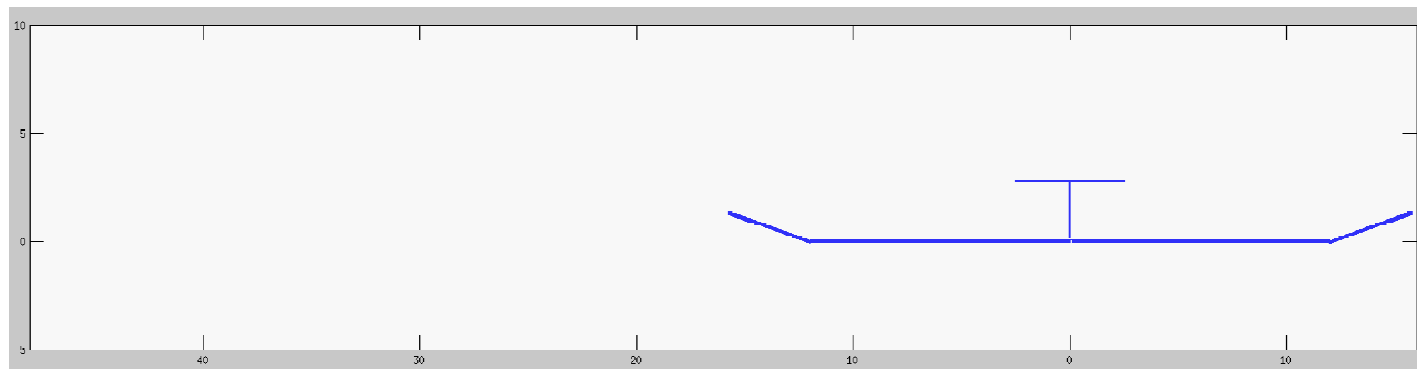
# Aerodynamic part of modeshapes



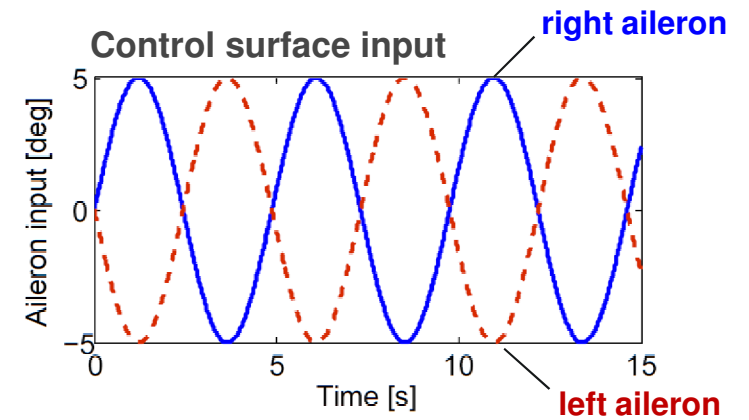
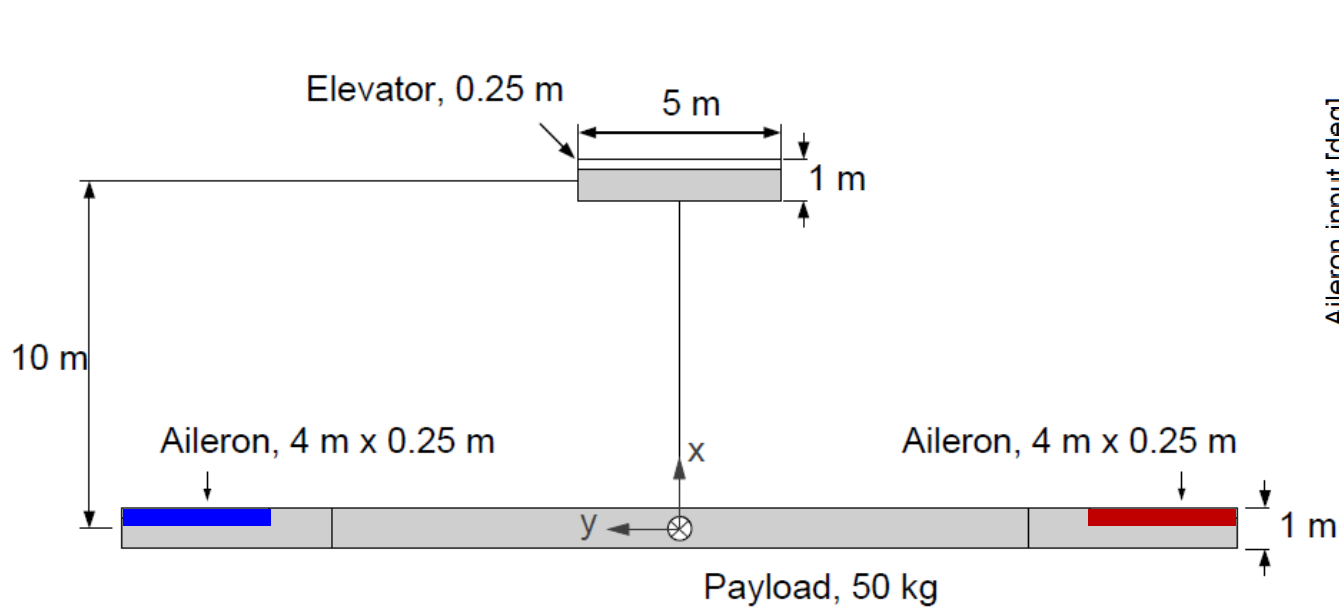
# Open-loop response of maneuvering HALE UAV



## Stiff configuration with $\sigma = 1000$

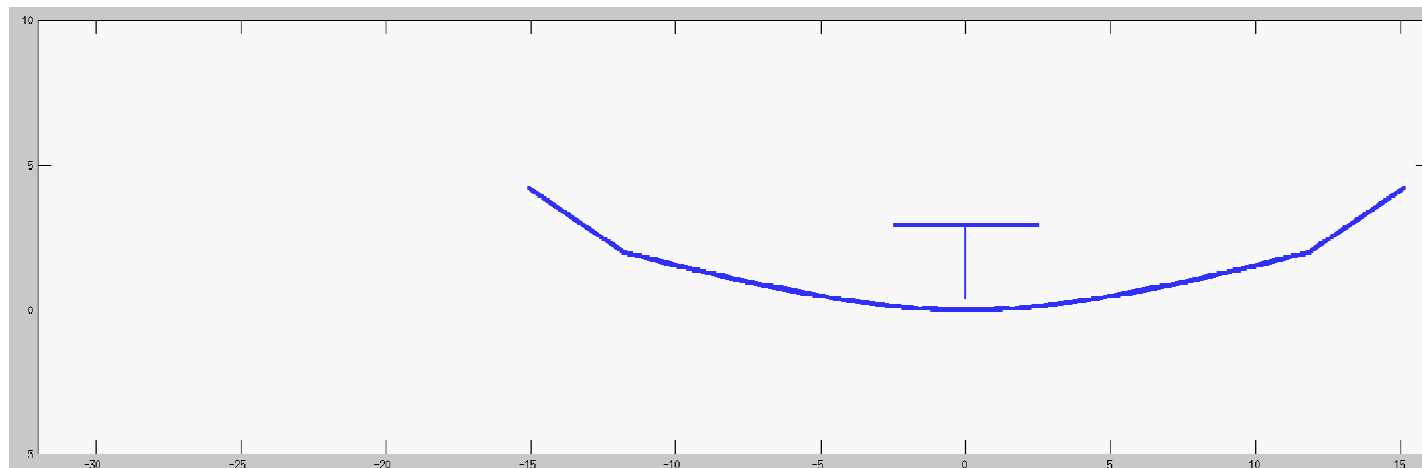


# Open-loop response of maneuvering HALE UAV



**System size:**  
**3239 aerodynamic states**  
**1068 structural states**

## Flexible configuration with $\sigma = 2$



# Aeroelastic system for maneuvering aircraft

## Linearised UVLM

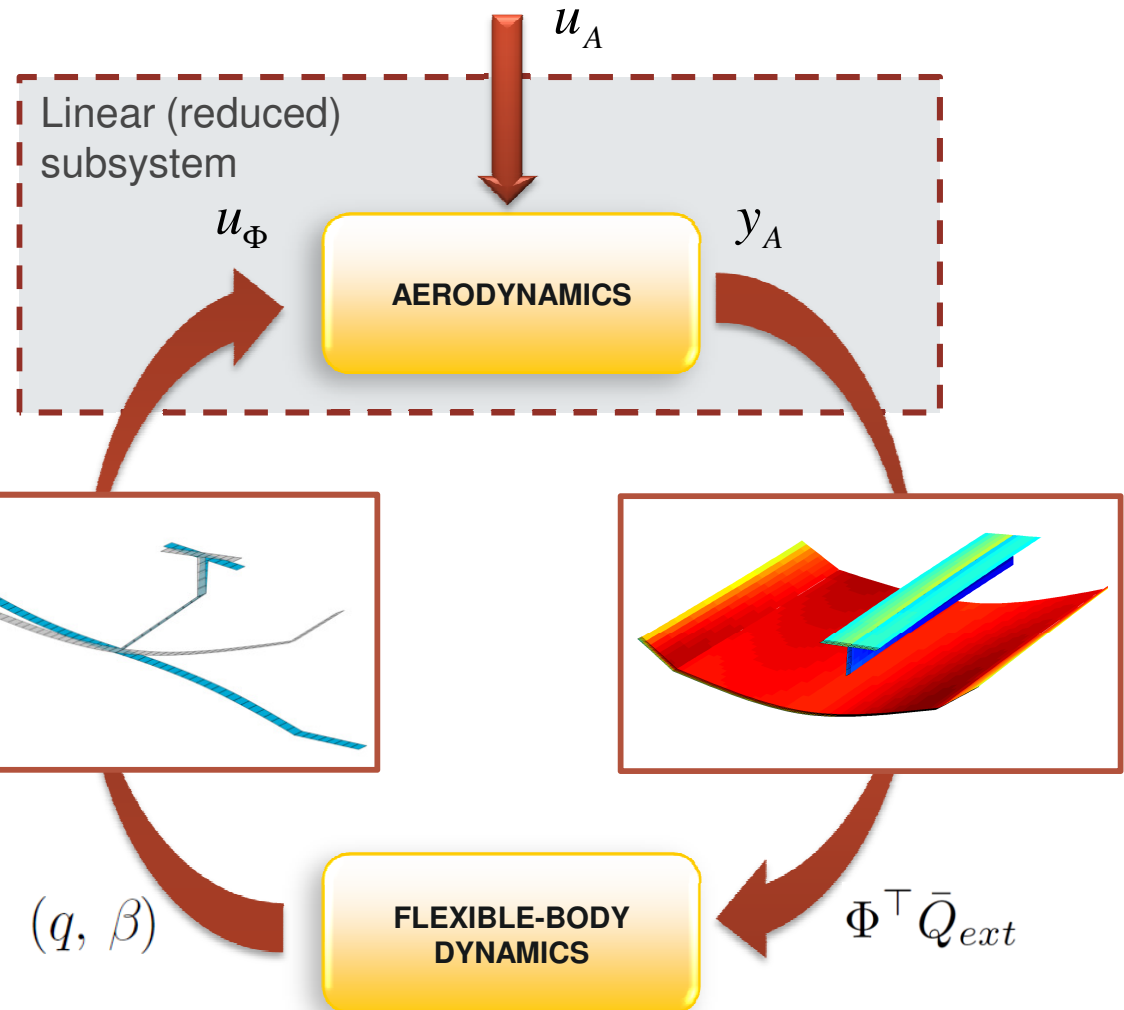
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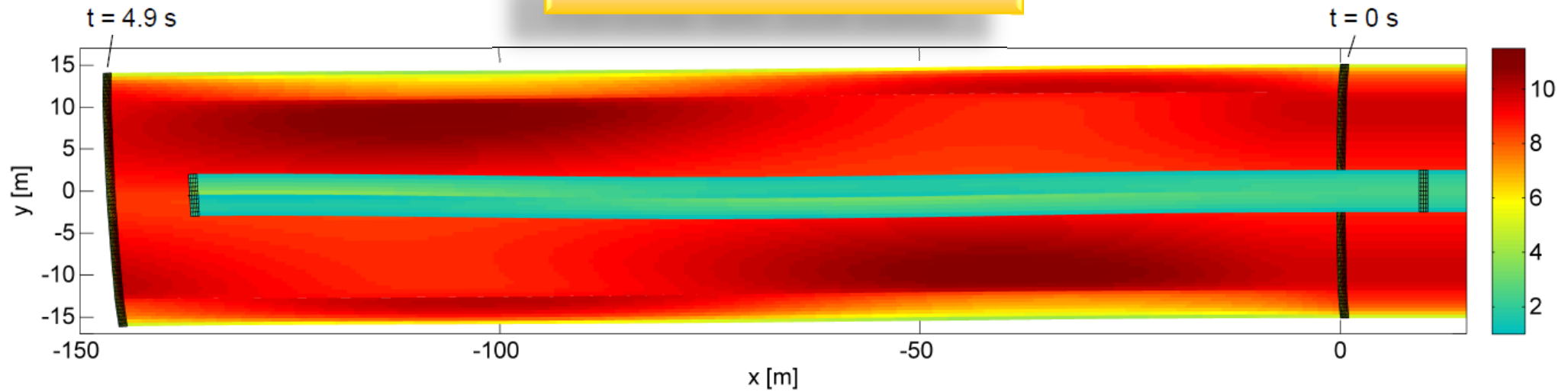
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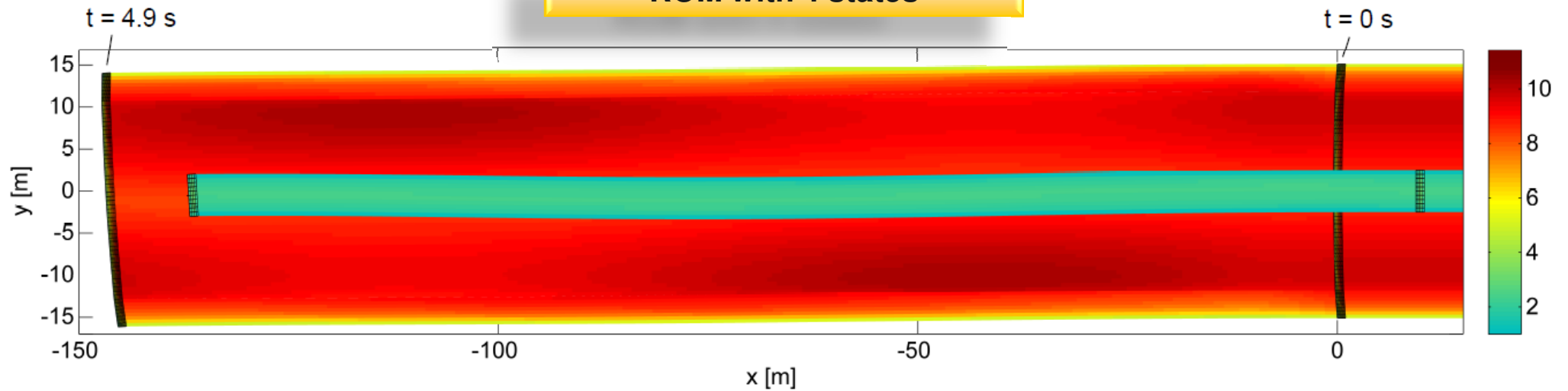


# ROM response of maneuvering HALE UAV

Full-order with 3239 states

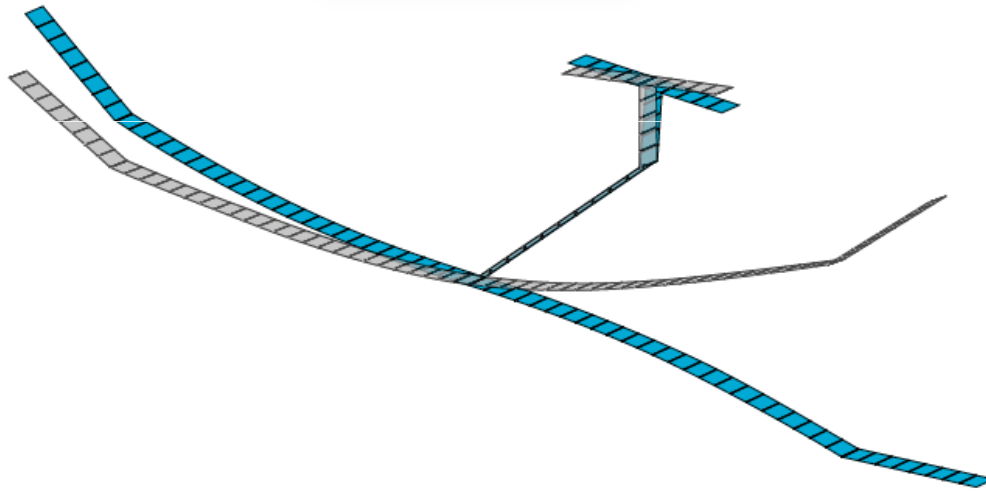


ROM with 4 states

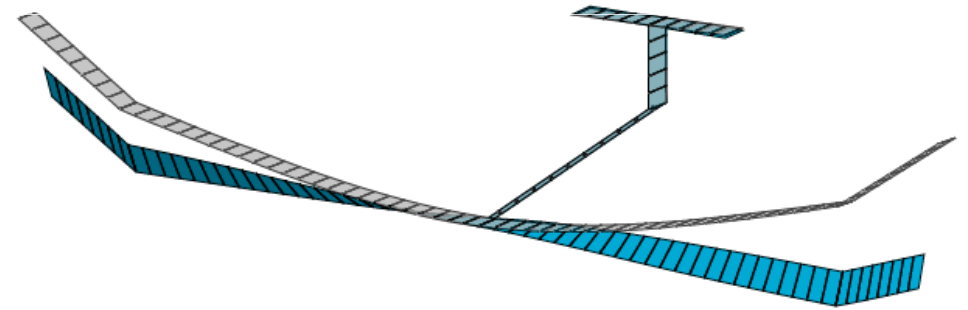


# ROM response of maneuvering HALE UAV

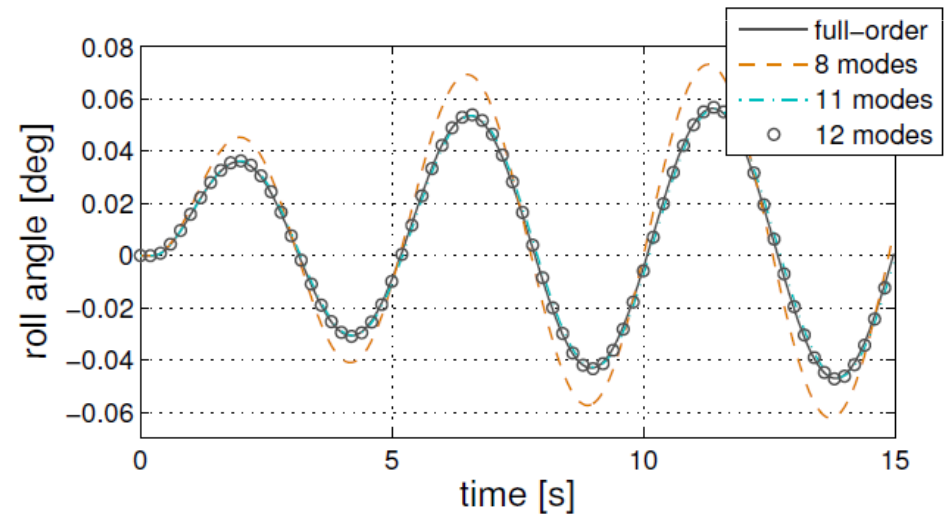
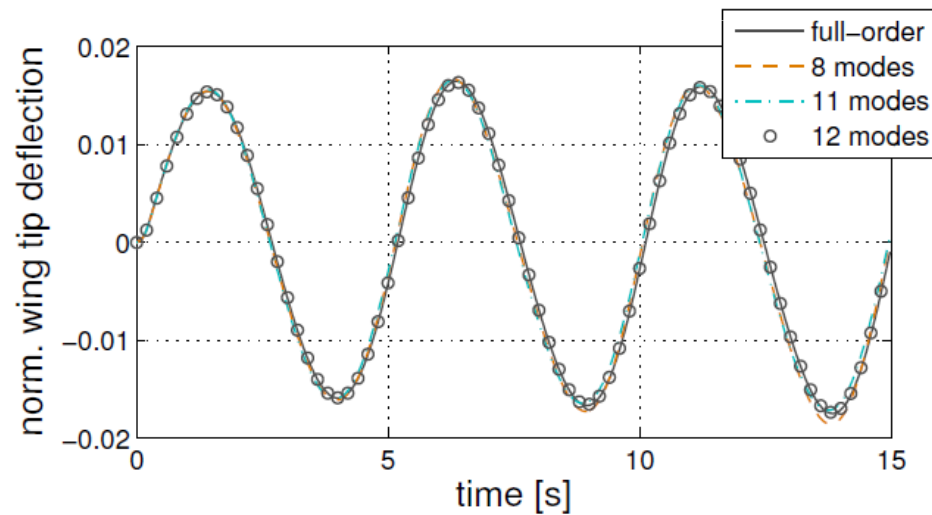
Mode 8



Mode 11

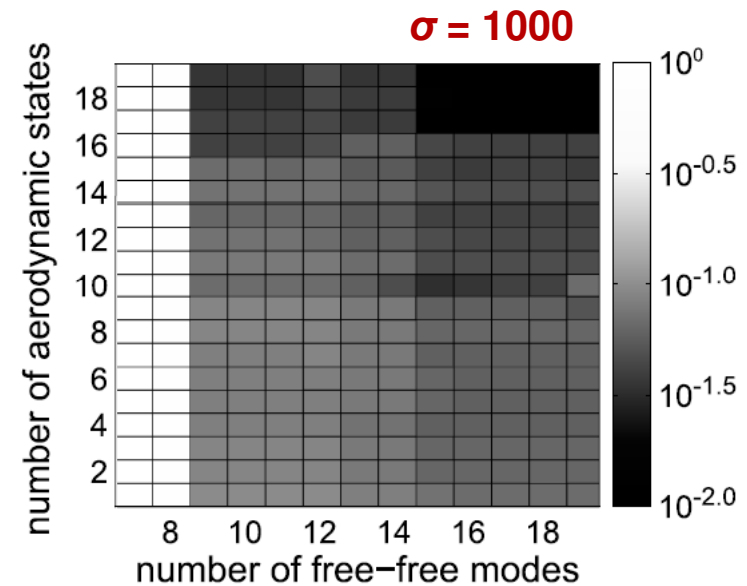
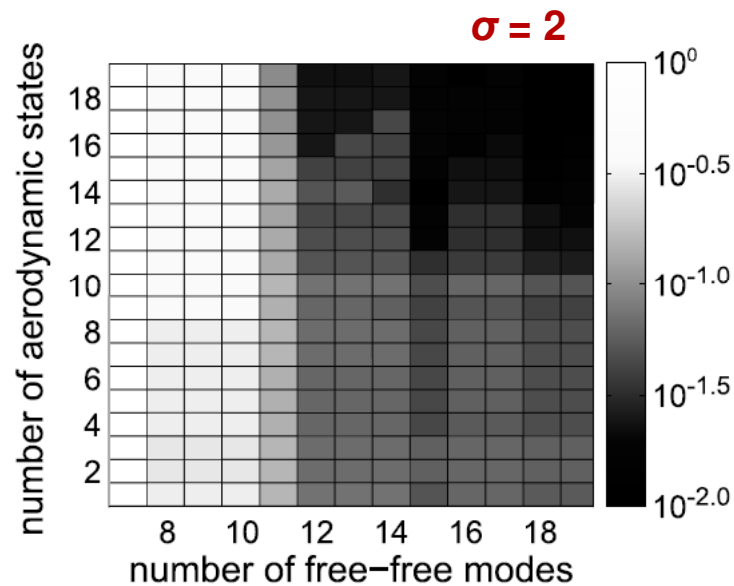


Reduce number of free-free modes



## Generic approach?

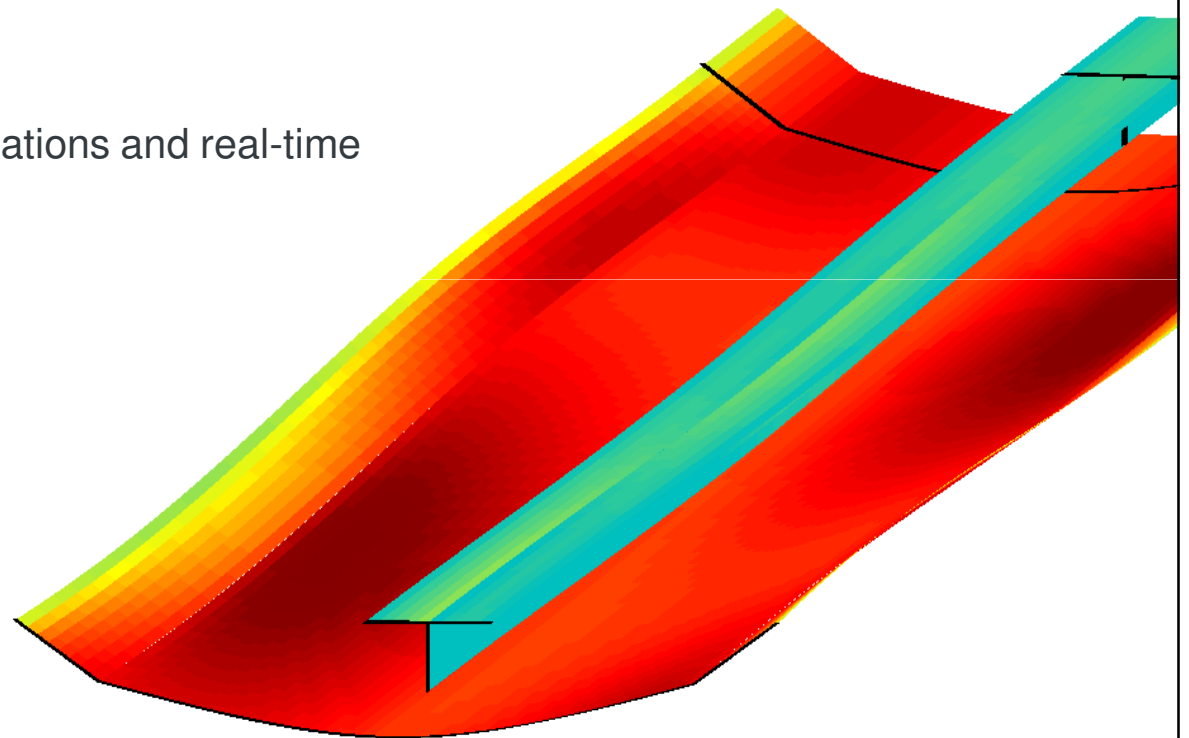
- $L_\infty$  error norm of flight dynamic response
  - Aeroelastic framework allows automatic exploration of ROM
  - Inclusion of unnecessary elastic modes harms the balanced truncation of the aerodynamic system
  - Generic approach for range of parameters, e.g. stiffness and flight speed



**Model reduction by three orders of magnitude!**

## Concluding remarks

- ROM of an integrated framework for the analysis of very efficient aircraft in time domain
- Approach provides an alternative to frequency-based methods at a similar system size, but:
  - large trim deformations
  - includes coupling effects between aeroelastic and rigid-body dynamics response
  - captures the unsteadiness of the 3D flow
- Ideal for robust control synthesis, load calculations and real-time simulations of next-generation aircraft
  - Golland wing
  - Representative HALE aircraft



# **Model Reduction in Flexible-Aircraft Dynamics with Large Rigid-Body Motion**

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Department of Aeronautics, Imperial College London

**Special thanks to EPSRC and Dr. Murua (University of Surrey)**