Matlab code for damping identification using energy balance technique

The Matlab code provided performs the identification method described in [1] using the data obtained from experiments. This document contains a brief description of the theory and the instruction to use the code for the test cases presented.

The energy method

The equations of motion of a damped multi degree-of-freedom system can be written in the matrix form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{D}\,\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}) = \mathbf{g}(t), \tag{1}$$

where $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the mass matrix, $\mathbf{K} \in \mathbb{R}^{n\times n}$ is the stiffness matrix, $\mathbf{D} \in \mathbb{R}^{n\times n}$ represents one of the possible damping matrices of coefficients multiplied by $\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}) \in \mathbb{R}^{n\times 1}$, a function of displacements, velocities or accelerations. $\mathbf{x} \in \mathbb{R}^{n\times 1}$ represents the vector of displacements and $\mathbf{g}(t) \in \mathbb{R}^{n\times 1}$ the excitation input vector. Premultiplying by $\dot{\mathbf{x}}^T$ and then integrating over time, the energy equation is obtained:

$$\int_t^{t+T_1} \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{M}\ddot{\mathbf{x}}\mathrm{d}t + \int_t^{t+T_1} \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{K}\mathbf{x}\mathrm{d}t + \int_t^{t+T_1} \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{D}\,\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}})\mathrm{d}t = \int_t^{t+T_1} \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{g}(t)\mathrm{d}t. \tag{2}$$

If the excitation force $\mathbf{g}(t)$ and the response $\mathbf{x}$ are periodic, the integration of conservative components of eq. (2) is zero over a full cycle of periodic motion. So if T is the period of $\mathbf{g}(t)$ and $\mathbf{x}$, the sum of kinetic and potential energy over this period is zero:

$$\int_t^{t+T} \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{M}\ddot{\mathbf{x}}\mathrm{d}t + \int_t^{t+T} \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{K}\mathbf{x}\mathrm{d}t = 0 \tag{3}$$

so eq. (2) becomes

$$\int_t^{t+T} \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{D}\,\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}})\mathrm{d}t = \int_t^{t+T} \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{g}(t)\mathrm{d}t. \tag{4}$$

Eq. (4) represents the balance between the energy dissipated by the damping mechanisms on the left hand side of the equation and the energy input to the system on the right hand side. This equation is the base of the energy-balance identification method. There are no restrictions on $\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}})$ and $\mathbf{M}$ and $\mathbf{K}$ are not required.

Damping-pattern matrix approach

The damping-pattern matrix approach is used to reduce the number of unknowns and to provide an easiest implementation in FEM models. A symmetric viscous damping matrix of a system with n degrees of freedom has $(n^2 + n)/2$ parameters to identify; this number can be considerably reduced by using information and engineering knowledge of the system under study. Starting from eq. (4), in the case of viscous damping and considering the initial instant t=0, it becomes

$$\int_0^T \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{C}\, \dot{\mathbf{x}}\, dt = \int_0^T \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{g}(t)\, dt \tag{5}$$

where $\mathbf{C}$ is the viscous damping matrix, which can be written as

$$\mathbf{C} = \sum_{i=1}^{p} c_i \mathbf{L_i} \tag{6}$$

where $\mathbf{L_i} \in \Re^{n \times n}$ is a matrix which indicates the location of the i^{th} of p different viscous damping sources of amplitude c_i.

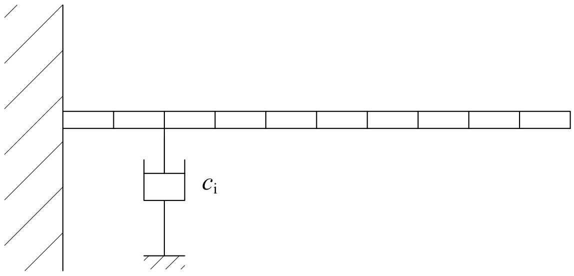

Figure 1: Absolute dashpot connecting DOF 2 to the ground

In the case of an absolute dashpot connecting one degree of freedom (e.g. degree-of-freedom 2, see figure 1) of the structure to the ground, $\mathbf{L_i}$ takes the form

$$\mathbf{L_i} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{7}$$

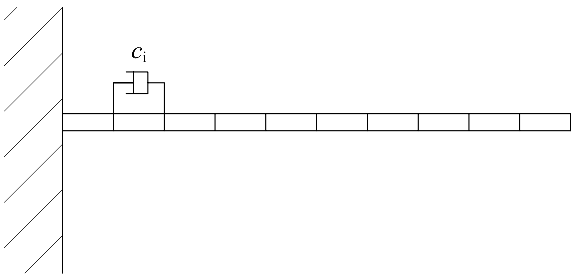

Figure 2: Relative dashpot connecting DOF 1 to DOF 2

In this case the pattern approach does not help the reduction of the number of unknowns, but helps a systematic procedure to define the damping sources in an automated way.

In the case of a relative dashpots connecting two degrees of freedom together (e.g degree-of-freedom 1 and 2, see figure 2), $\mathbf{L_i}$ takes the form

$$\mathbf{L_i} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{8}$$

which allows the reduction of the number of parameters to identify from 4 to 1. If the damping between two consecutive degrees of freedom is assumed to be the same for all the different couples (figure 3) representing, for example, the material damping between identical

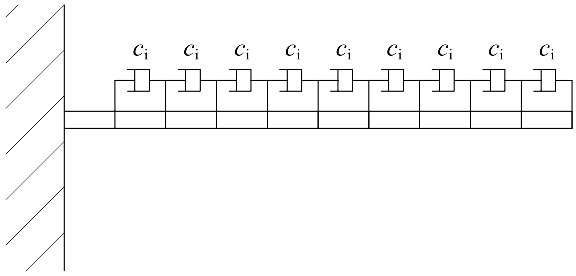

Figure 3: Identical relative dashpots connecting consecutive DOFs

elements or similar connections or joints between parts of the structure, $\mathbf{L_i}$ can take the form

$$\mathbf{L_i} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 \\ 0 & -1 & 2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \tag{9}$$

reducing the number of non-zero unknowns, in a 10 degrees of freedom example, from 28 to 1.

Assuming p different possible configurations for the damping sources, the energy equation (5) can be arranged as

$$c_1 \int_0^T \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{L_1} \, \dot{\mathbf{x}} \, dt + c_2 \int_0^T \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{L_2} \, \dot{\mathbf{x}} \, dt + \ldots + c_p \int_0^T \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{L_p} \, \dot{\mathbf{x}} \, dt = \int_0^T \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{g}(t) \, dt \tag{10}$$

By exciting the structure with q excitations at different frequencies, different versions of eq. (10) are obtained and arranged in a matrix form

$$\begin{bmatrix} \int_0^{T_1} \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{L_1} \, \dot{\mathbf{x}} \, dt & \ldots & \int_0^{T_1} \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{L_p} \, \dot{\mathbf{x}} \, dt \\ \vdots & \vdots & \vdots \\ \int_0^{T_q} \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{L_1} \, \dot{\mathbf{x}} \, dt & \ldots & \int_0^{T_q} \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{L_p} \, \dot{\mathbf{x}} \, dt \end{bmatrix} \begin{Bmatrix} c_1 \\ \vdots \\ c_p \end{Bmatrix} = \begin{Bmatrix} \int_0^{T_1} \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{g_1}(t) \, dt \\ \vdots \\ \int_0^{T_q} \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{g_q}(t) \, dt \end{Bmatrix} \tag{11}$$

or, in a more compact form,

$$\mathbf{A}\mathbf{c} = \mathbf{e} \tag{12}$$

Eq. (12) can be solved for vector $\mathbf{c}$ using least square techniques and forcing the non-negative definiteness of the identified damping matrix at the same time. When vector $\mathbf{c}$ is calculated, the full identified viscous damping matrix can be obtained from eq. (6)

Experimental setup

The experiment consists in an aluminium cantilever beam (660×40×4 mm) with several sources of damping attached at different locations (figure 4). The structure is excited with

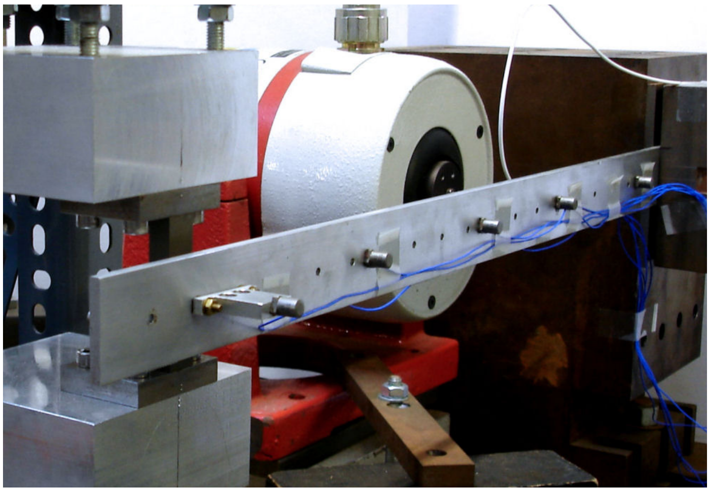

Figure 4: Experimental setup

a shaker in degree-of-freedom 3 (figure 5) and accelerations are measured in ten different locations equally spaced along the length of the beam. The structure is excited with several

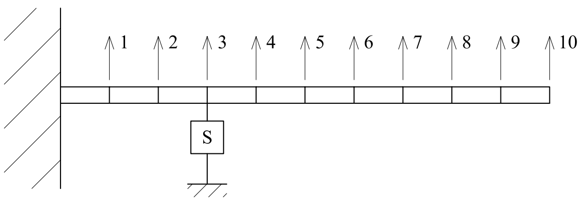

Figure 5: Definition of the DOFs of the experiment and location of the shaker

different single frequency excitations at frequencies close to the natural frequencies (where the damping is assumed to be more relevant) and the measured time histories are used to apply the identification method proposed.

Analytical integration of experimental data

A single frequency sinusoidal function has been used to curve-fit the experimental measurements in order to use an analytical integration of accelerations to obtain velocities and displacements and to obtain an analytical expression of the integrals present in eq. (11). Considering a single test with an excitation at frequency ω_i, vector $\mathbf{g_i}(t)$ take the form

$$\mathbf{g_i}(t) = \begin{Bmatrix} 0 \\ 0 \\ g_i(t) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{13}$$

The input is assumed to be perfectly sinusoidal, so the measurement of $g_i(t)$ from the force transducer is fit to a harmonic function (figure 6) as

$$g_i(t) = r_i \mathrm{sin}(\omega_i t) + s_i \mathrm{cos}(\omega_i t) \tag{14}$$

by estimating the two coefficients r_i and s_i using least squares technique.

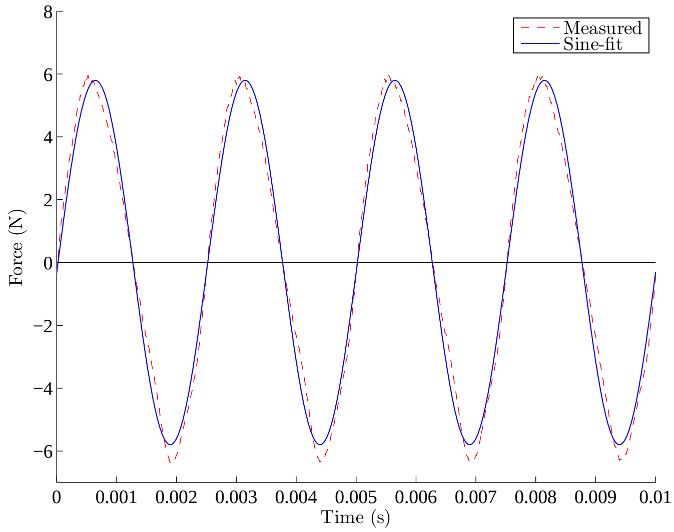

Figure 6: Typical measured and sine-fit force

The same procedure is used for the measurement from the ten accelerometers as

$$\ddot{\mathbf{x}}_{\mathbf{i}}(t) = \begin{Bmatrix} \ddot{x}_{1_i}(t) \\ \ddot{x}_{2_i}(t) \\ \ddot{x}_{3_i}(t) \\ \ddot{x}_{4_i}(t) \\ \ddot{x}_{5_i}(t) \\ \ddot{x}_{6_i}(t) \\ \ddot{x}_{7_i}(t) \\ \ddot{x}_{8_i}(t) \\ \ddot{x}_{9_i}(t) \\ \ddot{x}_{10_i}(t) \end{Bmatrix} = \begin{Bmatrix} u_{1_i} \\ u_{2_i} \\ u_{3_i} \\ u_{4_i} \\ u_{5_i} \\ u_{6_i} \\ u_{7_i} \\ u_{8_i} \\ u_{9_i} \\ u_{10_i} \end{Bmatrix} \sin(\omega_i t) + \begin{Bmatrix} v_{1_i} \\ v_{2_i} \\ v_{3_i} \\ v_{4_i} \\ v_{5_i} \\ v_{6_i} \\ v_{7_i} \\ v_{8_i} \\ v_{9_i} \\ v_{10_i} \end{Bmatrix} \cos(\omega_i t) \tag{15}$$

or

$$\ddot{\mathbf{x}}_{\mathbf{i}}(t) = \mathbf{u}_{\mathbf{i}}\sin(\omega_i t) + \mathbf{v}_{\mathbf{i}}\cos(\omega_i t) \tag{16}$$

so that velocities and displacements can be calculated by analytical integration as

$$\dot{\mathbf{x}}_{\mathbf{i}}(t) = \frac{1}{\omega_i}\left(-\mathbf{u}_{\mathbf{i}}\cos(\omega_i t) + \mathbf{v}_{\mathbf{i}}\sin(\omega_i t)\right) \tag{17}$$

$$\mathbf{x}_{\mathbf{i}}(t) = -\frac{1}{\omega_i^2}\left(\mathbf{u}_{\mathbf{i}}\sin(\omega_i t) + \mathbf{v}_{\mathbf{i}}\cos(\omega_i t)\right) \tag{18}$$

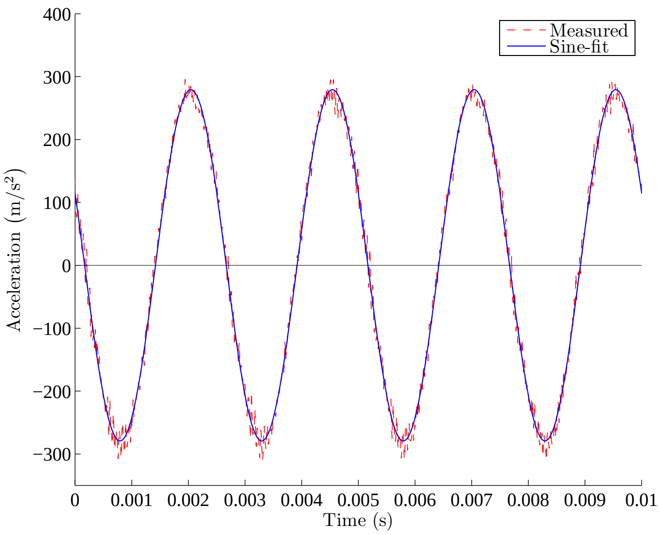

Figure 7: Typical measured and sine-fit acceleration, DOF 1

Using these expressions for forces and velocities allows the analytical calculation of the integrals in eq. (11) avoiding problems due to numerical integration and with the precise measurement of the period T too. For the case where the single frequency excitation is at frequency ω_i and all the measurements are fitted to harmonic functions at the same frequency only, the period T_i is simply

$$T_i = \frac{2\pi}{\omega_i} \tag{19}$$

The integrals present in eq. (11) become

$$\int_0^{T_i} \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{L_j} \, \dot{\mathbf{x}} \, dt = \frac{\pi}{\omega_i^3} \left(\mathbf{u_i}^{\mathrm{T}} \mathbf{L_j} \mathbf{u_i} + \mathbf{v_i}^{\mathrm{T}} \mathbf{L_j} \mathbf{v_i} \right) \tag{20}$$

and

$$\int_0^{T_i} \dot{\mathbf{x}}^{\mathrm{T}} \mathbf{g_1}(t) \, dt = \frac{\pi}{\omega_i^2} \left(v_{3_i} r_i - u_{3_i} s_i \right) \tag{21}$$

All the elements to solve equation (12) are now ready and easily obtainable from real experiments.

Test cases

Several experiments have been performed to validate the method proposed; the test cases presented include the undamped cantilever beam, the beam damped by an air dashpot and by a Coulomb friction device.

Case 1: undamped system

The first experiment is performed on the undamped cantilever beam in order to estimate the offset damping due to the clamp, material damping, cables, air and everything else. The chosen parameterisation for the equivalent viscous damping matrix is obtained by ten absolute dashpots as eq. (7) for the ten degrees of freedom to identify local sources of damping plus a matrix as eq. (9) for taking into account for the dissipations between consecutive degrees of freedom (assumed equal) for a total of 11 parameters to estimate.

Case 2: single air dashpot

The air dashpot used in the experiment consists in a cylinder with a moving piston (figure 8) which forces the air inside the cylinder to flow through an adjustable hole, allowing a variable damping coefficient. In this experiment the air dashpot has been set to a damping value of

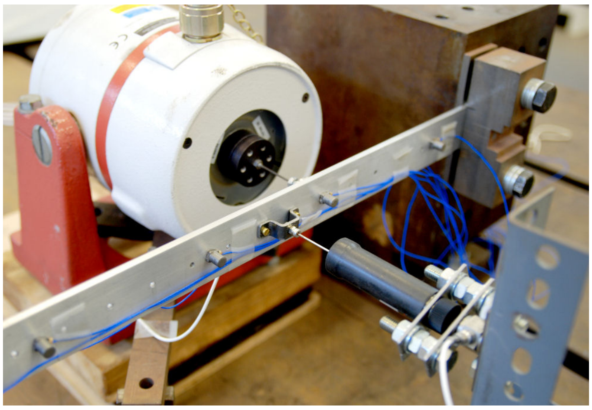

Figure 8: Air dashpot at DOF 4

approximately 5 Ns/m, located at degree-of-freedom 8.

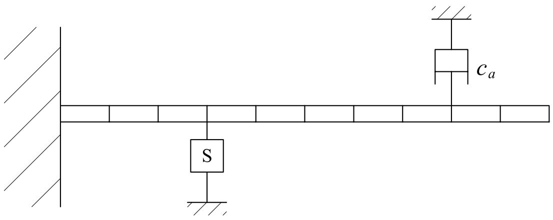

Figure 9: Location of the air dashpot

Case 3: coulomb friction

In the following experiment, a calliper acting on the aluminium wing as shown in figure 10 has been used. Different films of different materials can be applied to the wings and to the

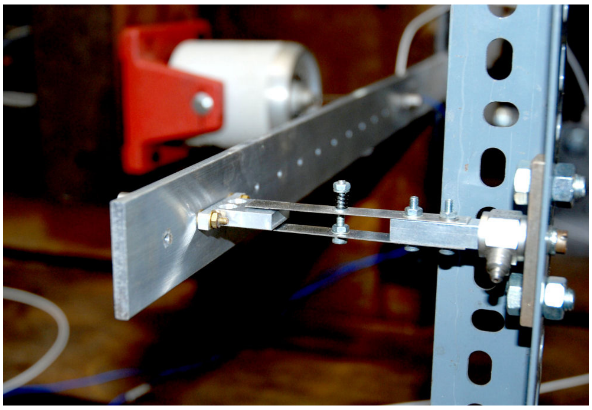

Figure 10: Coulomb friction device at DOF 9

callipers to provide different combination of materials and different friction coefficient μ. The normal force is provided by the screw in the middle of the calliper and it can be measured by static tests by a dynamometer applied in the contact point. The tangential force is measured by a force transducer located between the calliper and the support. By knowing the normal and tangential force is it possible to calculate the Coulomb friction coefficient μ to compare to the one obtained by damping identification. The Coulomb friction device has been attached

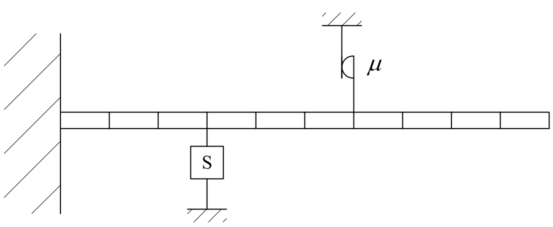

Figure 11: Location of the coulomb friction device

to degree-of-freedom 6.

Using the code

When the file "En_Method.m" is run into Matlab, it will ask to select which case to open. The case files (*.zip) are compressed *.mat files which have to be uncompressed in order to be used. The *.mat files contain the measured accelerations of the ten accelerometers (accNdata) for the different excitations at frequencies (excfr), the time vector (timedata), the measured excitation forces (forcedata) and some information about the test case.
After the selection of the file, the code will filter the data according to the band defined by the two variables "lowestfrequency" and "highestfrequency". This filter became necessary when it was discovered that the standard accelerometers used were affected by a phase delay at low frequencies. Then, the acceleration and force measurements are fit to a harmonic function as described in the previous sections.
At this point the code will ask if the plot of the approximated functions versus the original data has to be displayed. By clicking "Yes", a few periods of each measurement will be displayed in separate plots (one for each degree of freedom) together with their approximations, in order to evaluate if the harmonic functions are sufficient to approximate the measured data.
The next step is to define the damping pattern. In this example the chosen parameterisation for the equivalent viscous damping matrix is obtained by ten absolute dashpots in order to identify local sources of damping plus a matrix as eq. (9) for taking into account for the dissipations between consecutive degrees of freedom (assumed equal), for a total of 11 parameters to estimate.
The matrix of integrals **A** and the energy vector **e** are then computed using eqs. (20) and (21) and the energy equation is simply solved using the command "lsqnonneg" forcing the non-negativeness of the damping parameters which guarantees the non-negative definitess of the viscous damping matrix defined by eq. (6) using the damping pattern just defined.
A plot of the identified equivalent viscous damping matrix and a plot the energy contribution of each degree of freedom to the total dissipation of energy are then provided and the equivalent viscous damping matrix is saved in the variable "Cid".

Bibliography

[1] M. Prandina, J.E. Mottershead, and E. Bonisoli. Damping identification in multiple degree-of-freedom systems using an energy balance approach. *Journal of Physics: Conference Series*, 181(012006), 2009.