Synergy of Computational and Experimental Methods in the Development of an Inviscid Model for Insect-like Flapping Flight

Salman A. Ansari* Kevin Knowles† Rafał Żbikowski‡

Department of Aerospace, Power and Sensors
Cranfield University, Defence College of Management & Technology
Shrivenham, Swindon, SN6 8LA, England

s.a.ansari@cranfield.ac.uk

Abstract

This paper briefly discusses the aerodynamics of insect flight for the purposes of the design of insect-like flapping-wing micro air vehicles (MAVs). The kinematics of insect wings are described together with the associated nonlinear and unsteady aerodynamic phenomena. On this basis, the development of an aerodynamic modelling technique for insect-like flapping wings is discussed. This technique is analytical and results in exact but open-form wake-integrals which have to be solved numerically. Experimental verification is provided by suitable comparison with synthetic data generated using insect-like wing kinematics. The paucity of insect flight data is highlighted and the need for further integration of computational and experimental techniques in the development of models for insect-like flapping flight is identified. Some of the merits and challenges facing such a strategy are also discussed.

Keywords: *flapping-wing aerodynamics, insect flight, low Reynolds number flow, discrete vortex method*

1 Introduction

Insects are by far the commonest animal species around. Yet, several aspects of their livelihood have remained elusive over the ages, owing principally to their small size. One such aspect is their flight which has been described to a satisfactory degree only recently [1–3]. Due to the absence of 'roller bearings' in nature, insects are unable to resort to rotary mechanisms for flight and propulsion (cf. propellers and rotary wings). Instead, they rely on flapping motion, whereby they move their wings in a more or less reciprocating manner to generate sufficient forces (and moments) for flight. This is a concept they have perfected over time since they first began colonising the Earth over 300 million years ago [4, p. 262] and have established themselves as very successful flyers. They are also generally capable of hover, although most choose not to do so. It is this characteristic of insect flight from which we draw our inspiration.

In particular, insects exhibit remarkable agility at low speeds, they can hover stably, and take-off and land vertically, achieving all this in a very power-efficient manner. These characteristics make insect flight attractive for indoor flight, especially in confined, cluttered spaces. Agile flight inside buildings, caves and tunnels is of significant value in both military and civilian circles. Micro air vehicles (MAVs) with such characteristics show particular

*Post-Doctoral Research Officer
†Head, Aeromechanical Systems Group
‡Principal Research Officer

promise in so-called D^3—dull, dirty and dangerous—environments. Current surveillance assets (e.g. satellites, UAVs), however, possess virtually no capabilities of information-gathering inside buildings. The focus on indoor flight leads to the requirement of a distinct flight envelope, including small size (ca 150 mm), hovering capability, high manoeuvrability and small acoustic signature, amongst other things. Research at Cranfield University's Shrivenham campus has, therefore, been aimed at these insect-like flapping-wing MAVs (or FMAVs) [5–10].

2 Flapping-Wing Problem

2.1 Kinematics

Although insect flight kinematics have been observed as early as the advent of the film camera in the 19th century [11], it is only since the relatively recent availability of high-speed photography that they have been described in any sufficient detail [12–14]. Despite these recent advances, such data are still relatively scarce [15].

Insects have either one or two pairs of wings. We restrict ourselves to one pair of wings—so-called *Diptera*—and the description that follows is for such an insect. *Diptera* make very efficient flyers and, in particular, can hover stably. The overall flapping motion consists essentially of three component motions—sweeping (fore and aft motion), heaving/plunging (up and down movement) and pitching (varying incidence)—much like the sculling motion of the oars on a rowboat. Flapping frequency is typically in the range 5–200 Hz.

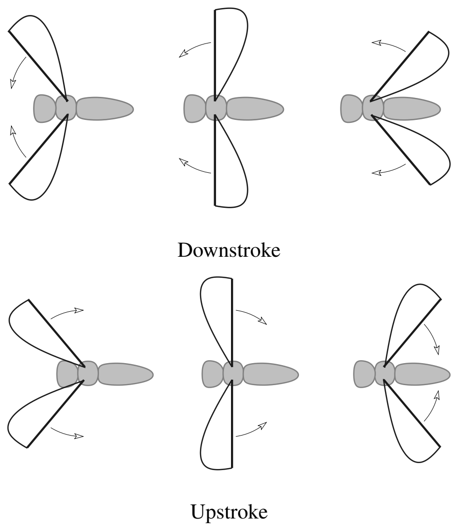


Figure 1: Top view of insect flapping

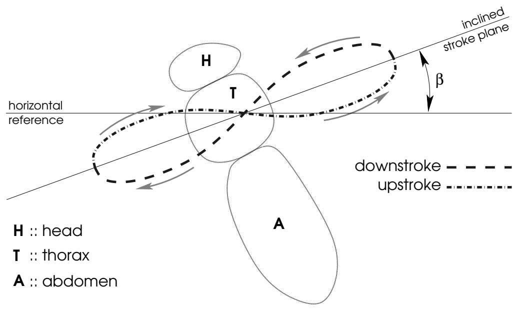


Figure 2: Side view of insect flapping

The wing motion can be divided broadly into two phases — *translational* and *rotational*. The translational phase consists of the *downstroke*—the motion of the wing from its rearmost to its foremost position (relative to the body)—and the *upstroke*, which describes the return cycle (see Figure 1). At either end of these halfstrokes, the rotational phases—so-called *stroke reversals*—come into play, whereby the wings rotate rapidly and reverse direction for the subsequent halfstroke. During this process, the morphological lower surface becomes the upper surface so that the leading edge always leads (Figure 1). The path traced out by the wing tip (relative to the body) during the wing stroke is not dissimilar to a figure-of-eight on a spherical surface (as the wing semi-span is constant; see Figure 2). The wing flaps back and forth about a roughly constant plane called the *stroke plane* (analogous to the tip-path-plane for rotorcraft).

During a halfstroke, the wing accelerates to a roughly constant speed around the middle of the halfstroke, before slowing down to rest at the end of it. The velocity during the wingbeat cycle is, therefore, non-uniform and for hover, in particular, the motion of the wing tip does not vary dramatically from a pure sinusoid [12]. Wing pitch also changes during the halfstroke, increasing gradually as the halfstroke proceeds.

2.2 Aerodynamics

Owing to the continually-varying nature of the kinematics in insect flapping flight, it comes as no surprise that the associated fluid flow behaves markedly differently from that observed in conventional fixed- or rotary-wing flight. Since each half-cycle starts from rest and comes to a stop, the velocity distribution during the flapping motion is non-uniform making the resulting airflow complex. As a result, the aerodynamic force varies in amplitude and direction during each wingbeat cycle. The variability of the force is compounded by the strong influence of the viscosity of air (due to the small scale) and significant interaction of the wing with its own wake (especially in hover). There is also the further aeroelastic complication resulting from significant fluid-structure interactions.

The flow associated with insect flapping flight (and scales pertaining to micro air vehicles) is incompressible, laminar, unsteady and occurs at low Reynolds numbers. Despite their short stroke lengths and small Reynolds numbers, insect wings generate forces much higher than their quasi-steady equivalents. Dudley & Ellington [16] found that for bumblebees, the quasi-steady estimates of lift and power requirements fail at all flight speeds. In a similar study by Wakeling & Ellington [17], the mean lift coefficient required for flight for the dragonfly *Sympetrum sanguineum* and the damselfly *Calopteryx splendens* was 'reverse-engineered' and, in both cases, was found to exceed the maximum possible under quasi-steady conditions. These observations strongly indicate the presence of unsteady aerodynamic phenomena.

Insect flapping flow is now understood to comprise two components — *attached* and *separated* flow [15]. The attached flow refers to the *freestream* flow on the aerofoil as well as that due to its *unsteady motion* (sweeping, heaving and pitching). For insect-like flapping wings, flow-separation is usually observed at both leading and trailing edges — the *leading-edge vortex*, which is bound to the wing for most of the duration of each halfstroke, and the *trailing-edge wake* that leaves smoothly off the trailing edge. Flow is more or less attached in the remaining regions of the wing.

The leading-edge vortex is now believed to be responsible for the augmented forces observed [18, 19]. It starts close to the wing root and spirals towards the tip where it coalesces with the tip vortex and convects into the trailing wake [18, 20]. The overall structure of the leading-edge vortex has been likened to that observed on low-aspect-ratio delta wings [21,22]; it is produced and fed by a leading-edge separation. From the literature, it would be appear that the spanwise spiralling nature of the leading-edge vortex is a more pronounced feature in larger insects [18, 23] that operate at higher Reynolds numbers ($Re \sim 5000$). In their experiments on a dynamically scaled-up model of the much smaller fruit-fly *Drosophila melanogaster* ($Re \sim 200$), Dickinson *et al* [24, 25] have also observed a strong leading-edge vortex but with weak spanwise flow.

As flapping wings move through the air, they encounter *apparent mass* forces arising from the mass of surrounding air also set in motion due to the wing movement, particularly due to the high accelerations and rapid stroke reversals. Because of the repeated accelerations of the wing and the brevity of the halfstrokes, starting vortices are created and remain in the vicinity of flapping wings, having a hindering effect on the growth of lift on the wings — the so-called *Wagner effect* [26]. In hover and slow forward flight, flapping wings are also likely to be affected by the *returning wake* from previous wingbeats [14, 27]. The importance of this 'wake capture' has also been noted by Grodnitsky & Morozov [28] who suggested that insects and birds have special mechanisms whereby they extract energy back from their near vortex wake. A similar view was expressed by Ennos [29] who speculated that, in flies, the kinematics were helped by the aerodynamics.

3 Aerodynamic Model

Aerodynamic analysis of insect flapping-wing flight has received relatively little attention until recently [1–3]. Even then, most models come from the biological community and as such are predominantly simple quasi-steady approaches [30–33]. Computational fluid dynamics (CFD) techniques are generally more detailed and can offer higher accuracy [19,34,35] but come at the cost of high CPU power and time. There is also the additional difficulty of generating dynamically-changing moving meshes. To model the flapping-wing problem more accurately than quasi-steady methods and more rapidly than conventional CFD, an aerodynamic modelling approach that captures the important features of the flight regime is required. A number of analytical approaches can be found in the literature but they either tackle a somewhat different problem [36] or the models are lacking in some respects [37, 38].

3.1 Modelling Methodology

As part of our study on flapping-wing micro air vehicles, a nonlinear, inviscid unsteady aerodynamic model was developed [39, 40, for full details, see [41]]. The model was required to capture the most important physical characteristics of the flow regime (identified above) and, in particular, the positions of flow-separation lines. The main output was a tool capable of predicting, with reasonable accuracy, the forces and aerodynamic moments experienced by an insect-like flapping wing. The circulation-based model is based [15] broadly on the seminal work of von Kármán & Sears [42], with refinements along the lines of McCune *et al* [43] and McCune & Tavares [44] introduced albeit with several significant extensions. The model itself is quasi-three-dimensional; strip theory is used to divide the wing spanwise into chordwise sections that are each treated essentially as two-dimensional. As also noted by [45], the low aspect ratio and high solidity[1] of insect wings requires that *radial chords* be used instead of normal (straight) chords (see Figure 3). This is necessary because each wing section would otherwise see a curved (and significant) incidence velocity. As a result, each wing section resides in a radial cross-plane that is then unwrapped flat and the flow is solved as a planar two-dimensional problem. The overall effect on the wing is obtained by integrating along the span.

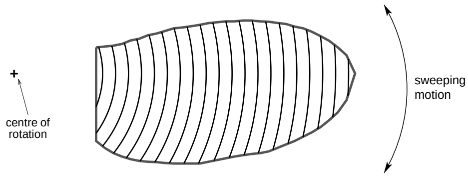


Figure 3: Radial chord representation of a fruit-fly wing

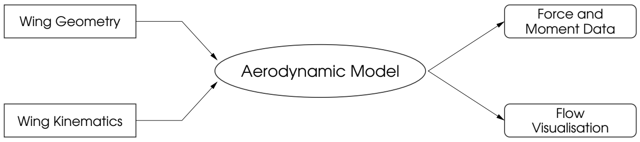


Figure 4: Data flow in the aerodynamic model

A number of simplifying assumptions are made. Potential-flow (inviscid) methods are used and flow-separation from both leading and trailing edges is modelled by enforcing the Kutta-Joukowksi condition at the points of wake-inception. Further, the flow is assumed to be irrotational (except at solid boundaries and discontinuities in the wake). Although viscosity is generally ignored, its effects are included indirectly in the form of the Kutta-Joukowksi condition and in the formation and shedding of vortices. The wing is taken to be a thin, rigid, flat plate and three-dimensional effects, due to tip vortices or interactions between adjacent wing sections, are ignored. Finally, the attached and separated flows are assumed to be linearly superposable (on the basis of the linearity of the underlying Laplace's equation).

As a result, the problem is divided into two distinct components: a wake-free (quasi-steady) element and a wake-induced (unsteady) one. By satisfying the zero-through-flow (impermeability) condition at the wing surface and observing Kelvin's law that the total circulation in a control volume must remain constant, two coupled, nonlinear integral equations are arrived at. These novel equations are exact and describe the entire flow within a cross-plane (within the limits of the assumptions made). They are, however, nonlinear and as such do not have a closed form. Solutions are, therefore, found numerically.

3.2 Computational Implementation

The solution is implemented using the discrete point vortex method. A time-marching algorithm is used and implemented in Fortran (see [41] for full details). The aerofoil in each 2-D section is represented by a distribution of bound vortices and the zero-through-flow condition is enforced on its surface. The two wakes shed from the leading and trailing edges are also distributions of vorticity but these are free to move with the fluid flow. At each time-step, the quasi-steady bound circulation is computed for smooth flow at the trailing edge. Two new vortices are then released, one each from the leading and trailing edges, and placed such that they follow the trace left by

[1] Solidity refers to the ratio of wing planform area to area swept by the wing.

the previous vortex. The nonlinear equations referred to above are then solved simultaneously for the circulation strengths of the two new vortices.

At the end of the time-step, the solution is marched forward in time by convecting the shed vortices in the wake using a forward Euler scheme. During the more acute phases of the flapping cycle (e.g. stroke reversals), the time-steps are subdivided into finer sub-time-steps to give better resolution but at the cost of increased CPU time. A spin-off of this method is that flow-visualisation is automatically generated (see Figure 4).

Forces are computed by Kelvin's method of impulses [42, 46, 47]. The bound and shed vortices constitute vortex pairs that impart impulses between them. The combined time-rate-of-change of impulse of all vortex pairs is a measure of the force on the wing (since only the bound vortices sustain Kutta-Joukowski forces). Moment is computed similarly from the moment of impulse.

3.3 Experimental Validation

The validity of the theoretical model described above can only be established by comparing with experimental data. As a result of the formulation of the model, both force (and moment) data as well as flow-visualisation could be generated for comparison (Figure 4). As also noted by [15], experimental data in this field are rather scarce, owing principally to the size of insects and the difficulty of 'integrating' them into experimental setups. Most data in the field, therefore, have been generated from synthetic experiments [18, 20–22, 48–51]. We used two such datasets to make flow-visualisation [50] and force [52] comparisons (full details in [40, 41]).

3.3.1 Comparison with Dickinson and Götz [50]

Dickinson and Götz conducted their experiment in a glass aquarium filled with a 54% sucrose solution [50]. A rectangular wing (with 5 cm chord and 15 cm span) was traversed in a straight line at a constant angle of attack between a pair of baffles to limit any 3-D flow. The wing was impulsively started and then dragged at constant speed before being brought to an abrupt stop about 7.5 chord lengths later. The experiment was run at Reynolds number of 192, based on chord. The conditions for the experiment were reproduced and the flow solved for using the potential-flow model described above. A comparison with Dickinson's and Götz's experiment with the wing at 45° angle of attack was made and the results are described now (see Figure 5).

As the figure shows, there is remarkable agreement between the real, viscous experiment and the inviscid theoretical prediction, despite the very low Reynolds number. There exist a large number of fluid flows that are affected by viscosity to the first order and yet their motions do not show signs of restrictive amounts of viscous dissipation. Such flows may be treated as special cases of potential or irrotational flows, allowing them to be solved by well-known analytical methods (e.g. Hele-Shaw flows). The flow-visualisation photographs were for fixed instances at 1 through 4 chord lengths of travel since impulsive start. The theoretical prediction even captures the triangular structure on the right-hand side of the rolled-up end of the distorted leading-edge vortex after 4 chord lengths of travel (see Figure 5(d)), hence showing particular promise for the approach.

3.3.2 Comparison with Dickinson [52]

The force comparison is based on data provided by Dickinson [52] from the setup that was used in experiments by Dickinson *et al* [20, 24]. A scaled-up model of the wing of the fruit fly *Drosophila* dubbed the *Robofly* was used in which the wing executed an insect-like flapping motion at a frequency of about 0.17 Hz with the wing tip tracing out a flat figure-of-eight. The wing swept a semicircular (180°) arc, this being possible because only one wing was used (no mechanical interference from the 'other' wing). The experiment was conducted in a tank of mineral oil (density 880 kg/m^3) so Reynolds number was 160 based on mean chord and mean tip speed. Data were provided for 4 complete cycles (or 8 halfstrokes) starting from rest. The wing sweeping velocity is constant for most of the halfstroke but reverses direction at each stroke reversal. Similarly, angle of attack also remains more or less constant at 45° for most of the halfstroke but goes through a 90°-rotation at stroke reversal.

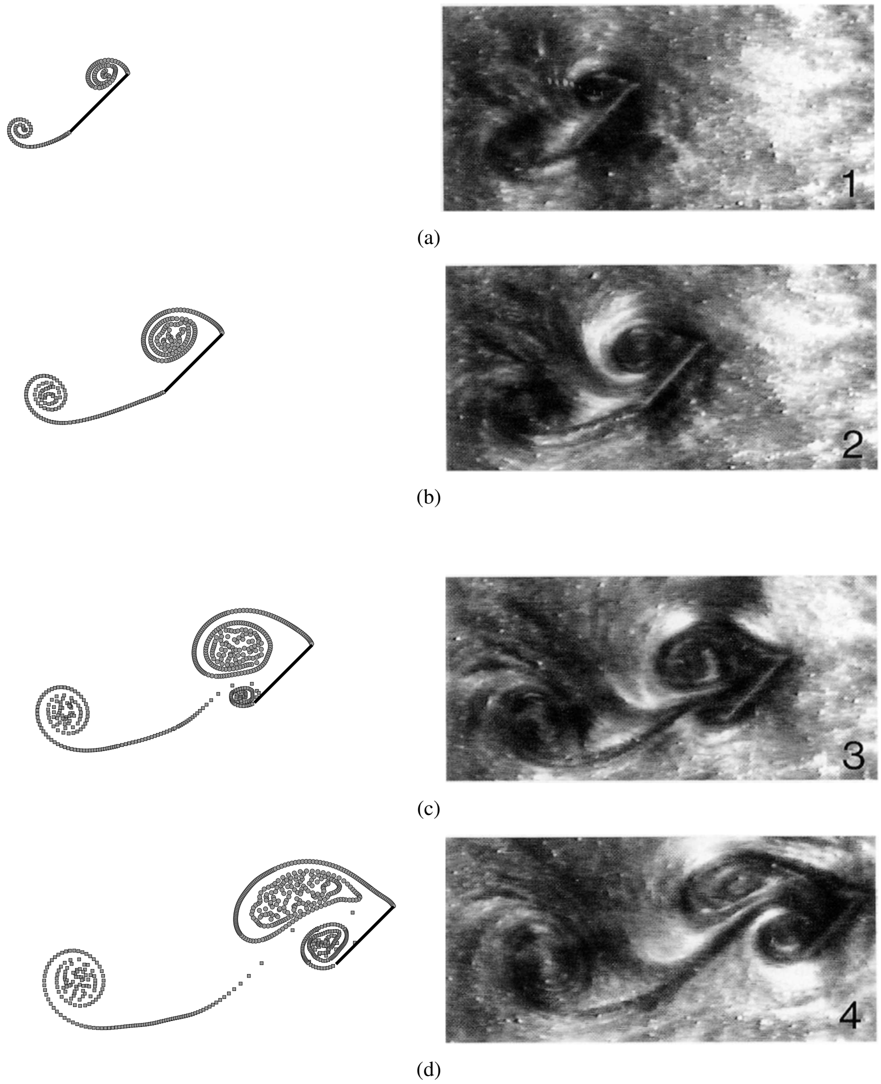
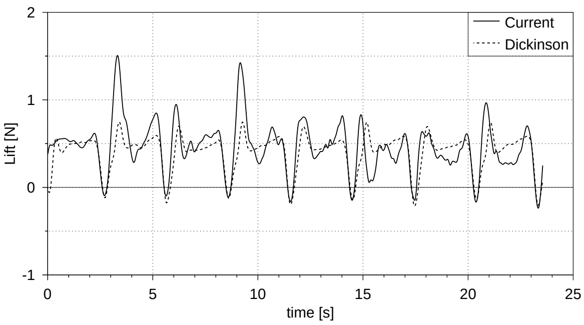
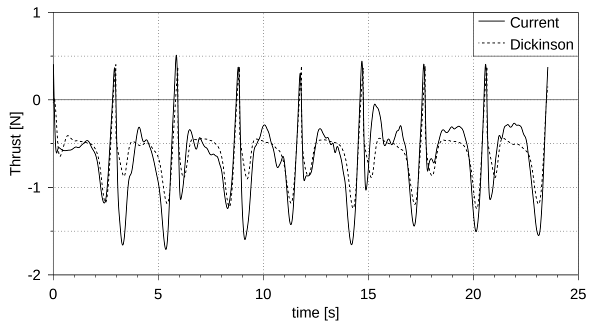


Figure 5: Comparison of flow-visualisation results from the current inviscid theoretical model (left) with the [50] experiment at $Re = 192$ (right). The numbers 1 through 4 refer to the number of chord lengths travelled since impulsive start



(a) Lift comparison



(b) Thrust comparison

Figure 6: Comparison of force data for Robofly data from [52] with results from the current theoretical simulation

Results from this force-comparison are shown in Figure 6. The forces on the flapping wing used for comparison were the vertical force opposing gravity (denoted lift) and the orthogonal force always in the direction of travel (denoted thrust). The similarity between the experimental data and the results of the simulation is immediately apparent. The pair of opposite spikes at stroke reversals are particularly well-captured by the theoretical model, occurring at the same points in time without any significant lag for both lift and thrust, thus accounting very well for unsteadiness in the flow. Moreover, the negative spikes for lift (and positive spikes for thrust) are nearly perfectly predicted in magnitude as well. The opposite spikes, however, are overestimated.

3.4 Extension of the Model

Despite the promising results produced by the unsteady, potential-flow model described above, it remains inviscid and is, therefore, subject to the associated limitations. The model is also quasi-3-D as 'three-dimensionality' is achieved simply by enforcing a limited span. In addition, any spanwise and tip-vortex effects are not handled in the current formulation. Extending the model to a higher order is, therefore, highly desirable.

With the rise in computing power, RANS-type Navier-Stokes calculations have become very competitive. Relatively quick 2-D DNS computations should also be possible which would offer further verification of our model. Dynamic meshing (collapsing and extending computational domain) in the vicinity of the flapping wing, especially in 3-D, still remains a challenge [53] but is likely to be overcome in due course (using, for example, overset grids [54]). Conventional CFD models are also well-poised to give novel insight into hitherto unexplored flow visualisation and aeroelastic phenomena.

As mentioned earlier, experimental data in this relatively novel field are still scarce and this, in part, has been responsible for hindering the development of suitable theoretical models for insect flight. There is a dire need for such experimental data. However, until such time, computational techniques are likely only to give predictions for 'insect-like' flapping flight. As more data become available, their integration with theoretical models will become increasingly important, especially as the modelling techniques improve. This will impact both analytical modelling techniques as well as grid-based CFD methods.

4 Conclusion

The development of a nonlinear, unsteady inviscid model for insect-like flapping wings has been described. The flapping-wing problem is introduced by means of its kinematics and pertinent aerodynamics. With this premise, a blade-element-type modelling methodology is described, incorporating the concept of 'radial chords' for extension to 3-D. The mathematical formulation of the problem eventually results in two coupled, nonlinear wake-integral equations which are then solved numerically using a forward Euler algorithm.

The main outputs from the theoretical model are forces (and moments) and flow-visualisation. These are used for experimental validation of the model and good agreement is found both in terms of force-prediction and flowfield representation. Improvements for the model are then considered and these are compared with the competitiveness of conventional CFD methods. The paucity of experimental data in the field is identified and its importance in further improving the computational approaches to flapping-wing modelling is discussed.

In conclusion, it is apparent from the discussion above that further integration of experimental and computational methods is essential for the development of accurate tools for the prediction of the aerodynamics of insect-like flapping wings. This will pave the way for more advanced studies in the subject, such as the aeroelasticity of flapping wings. They will also offer better capability for parametric studies of flapping-wing parameters for MAV-design.

Acknowledgements

The authors are grateful to Prof. Michael Dickinson of the California Institute of Technology for providing the validation data used in this work. Thanks are also due to the EPSRC (through grant numbers GR/M78472/01 and GR/S23025/01) and the UK Ministry of Defence for supporting this work.

References

[1] S. P. Sane. The aerodynamics of insect flight. *Journal of Experimental Biology*, 206(23):4191–4208, December 2003.

[2] K. V. Rozhdestvensky and V. A. Ryzhov. Aerohydrodynamics of flapping-wing propulsors. *Progress in Aerospace Sciences*, 39(8):585–633, November 2003.

[3] F. O. Lehmann. The mechanisms of lift enhancement in insect flight. *Naturwissenschaften*, 91(3):101–122, March 2004.

[4] Robert Dudley. *Biomechanics (Structures & Systems): A Practical Approach*. Oxford University Press, 1992.

[5] R. Żbikowski. Flapping Wing Autonomous Micro Air Vehicles: Research Programme Outline. In *Fourteenth International Conference on Unmanned Air Vehicle Systems*, volume Supplementary Papers, pages 38.1–38.5, 1999.

[6] Rafał Żbikowski. Flapping Wing Micro Air Vehicle: A Guided Platform for Microsensors. In *Royal Aeronautical Society Conference on Nanotechnology and Microengineering for Future Guided Weapons*, pages 1.1–1.11, 1999.

[7] R. Żbikowski. Aerodynamics: Red admiral agility. *Nature*, 420(6916):615–618, 2002.

[8] R. Żbikowski, K. Knowles, C. B. Pedersen, and C. Galiński. Some aeromechanical aspects of insect-like flapping wings in hover. *Proceedings of the Institution of Mechanical Engineers Part G: Journal of Aerospace Eengineering*, 218(G6):389–398, 2004.

[9] R. Żbikowski, C. Galiński, and C. B. Pedersen. A four-bar linkage mechanism for insect-like flapping wings in hover: Concept and an outline of its realisation. *Journal of Mechanical Design*, 127(4):817–824, 2005.

[10] C. Galiński and R. Żbikowski. Insect-like flapping wing mechanism based on a double spherical Scotch yoke. *Journal of the Royal Society Interface*, 2(3):223–235, 2005.

[11] E. J. Marey. Determination experimentale du mouvement des ailes des insectes pendant le vol. *C. R. Acad. Sci. Paris*, 67:1341–1345, 1868.

[12] C. P. Ellington. The Aerodynamics of Hovering Insect Flight: III. Kinematics. *Philosophical Transactions of the Royal Society of London Series B*, 305:41–78, 1984.

[13] A. Roland Ennos. The Kinematics and Aerodynamics of the Free Flight of Some Diptera. *Journal of Experimental Biology*, 142:49–85, 1989.

[14] Michael H. Dickinson, Fritz-Olaf Lehmann, and Sanjay P. Sane. Wing Rotation and the Aerodynamic Basis of Insect Flight. *Science*, 284:1954–1960, 18 June 1999.

[15] Rafał Żbikowski. On aerodynamic modelling of an insect-like flapping wing in hover for micro air vehicles. *Philosophical Transactions of the Royal Society of London Series A*, 360:273–290, 2002.

[16] R. Dudley and C. P. Ellington. Mechanics of Forward Flight in Bumblebees. II. Quasi-Steady Lift and Power Requirements. *Journal of Experimental Biology*, 148:53–88, 1990.

[17] J. M. Wakeling and C. P. Ellington. Dragonfly Flight: III. Lift and Power Requirements. *Journal of Experimental Biology*, 200:583–600, 1997.

[18] Charles P. Ellington, Coen van den Berg, Alexander P. Willmott, and Adrian L. R. Thomas. Leading-edge vortices in insect flight. *Nature*, 384:626–630, 19/26 December 1996.

[19] Hao Liu, Charles P. Ellington, Keiji Kawachi, Coen van den Berg, and Alexander P. Wilmott. A Computational Fluid Dynamic Study of Hawkmoth Hovering. *Journal of Experimental Biology*, 201:461–477, 1998.

[20] James M. Birch and Michael H. Dickinson. The influence of wingwake interactions on the production of aerodynamic forces in flapping flight. *Journal of Experimental Biology*, 206:2257–2272, 2003.

[21] Linda J. Martin and Peter W. Carpenter. Flow-visualisation experiments on butterflies in simulated gliding flight. *Fortschritte der Zoologie*, 24(2/3):307–316, 1977.

[22] Coen van den Berg and Charles P. Ellington. The Three-Dimensional Leading-Edge Vortex of a "Hovering" Model Hawkmoth. *Philosophical Transactions of the Royal Society of London Series B*, 352(1351):329–340, 29 March 1997.

[23] Alexander P. Willmott, Charles P. Ellington, and Adrian L. R. Thomas. Flow Visualization and Unsteady Aerodynamics in the Flight of the Hawkmoth, Manduca sexta. *Philosophical Transactions of the Royal Society of London Series B*, 352:303–316, 1997.

[24] J. M. Birch and M. H. Dickinson. Spanwise Flow and the Attachment of the Leading-Edge Vortex on Insect Wings. *Nature*, 412(6848):729–733, 2001.

[25] Sanjay P. Sane and Michael H. Dickinson. The Control and Flight Force by a Flapping Wing: Lift and Drag Production. *Journal of Experimental Biology*, 204:2607–2626, 2001.

[26] Michael H. Dickinson. Unsteady Mechanisms of Force Generation in Aquatic and Aerial Locomotion. *Amer. Zool.*, 36:537–554, 1996.

[27] Michael H. Dickinson. The Effects of Wing Rotation on Unsteady Aerodynamic Performance at Low Reynolds Numbers. *Journal of Experimental Biology*, 192:179–206, 1994.

[28] Dmitry L. Grodnitsky and Pahvel P. Morozov. Vortex Formation During Tethered Flight of Functionally and Morphologically Two-Winged Insects, Including Evolutionary Considerations on Insect Flight. *Journal of Experimental Biology*, 182:11–40, 1993.

[29] A. Roland Ennos. Inertial and Aerodynamic Torques on the Wings of Diptera in Flight. *Journal of Experimental Biology*, 142:87–95, 1989.

[30] Torkel Weis-Fogh. Quick Estimates of Flight Fitness in Hovering Animals, Including Novel Mechanisms for Lift Production. *Journal of Experimental Biology*, 59:169–230, 1973.

[31] C. P. Ellington. The Aerodynamics of Hovering Insect Flight: IV. Aerodynamic Mechanisms. *Philosophical Transactions of the Royal Society of London Series B*, 305:79–113, 1984.

[32] P. J. Wilkin and M. H. Williams. Comparison of the instantaneous aerodynamic forces on a sphingid moth with those predicted by quasi-steady aerodynamic theory. *Physiol. Zool.*, 66:1015–1044, 1993.

[33] Sanjay P. Sane and Michael H. Dickinson. The aerodynamic effects of wing rotation and a revised quasi-steady model of flapping flight. *Journal of Experimental Biology*, 205:1087–1096, 2002.

[34] Z. Jane Wang. Two Dimensional Mechanism for Insect Hovering. *Physics Review Letters*, 85(10):2216–2219, 4 September 2000.

[35] Ravi Ramamurti and William C. Sandberg. Computational Study of 3-D Flapping Foil Flows. In *39th AIAA Aerospace Sciences Meeting and Exhibit*, number AIAA-2001-0605, Reno, Nevada, 8-11 January 2001. 8-11 January.

[36] Marvin A. Jones. The separated flow of an inviscid fluid around a moving flat plate. *Journal of Fluid Mechanics*, 496:405–441, 2003.

[37] F. O. Minotti. Unsteady two-dimensional theory of a flapping wing. *Physical Review E*, 66(051907):1–10, 2002.

[38] Christopher B. Pedersen. *An indicial-Polhamus model of aerodynamics of insect-like flapping wings in hover*. PhD thesis, Cranfield University (RMCS Shrivenham), 17 June 2003.

[39] Salman Ahmad Ansari, Rafał Żbikowski, and Kevin Knowles. A nonlinear unsteady aerodynamic model for insect-like flapping wings in the hover: Part I. Methodology and analysis. Under consideration for publication in Journal of Fluid Mechanics, 2005.

[40] Salman Ahmad Ansari, Rafał Żbikowski, and Kevin Knowles. A nonlinear unsteady aerodynamic model for insect-like flapping wings in the hover: Part II. Implementation and validation. Under consideration for publication in Journal of Fluid Mechanics, 2005.

[41] Salman Ahmad Ansari. *A Nonlinear, Unsteady, Aerodynamic Model for Insect-like Flapping Wings in the Hover with Micro Air Vehicle Applications*. PhD thesis, Cranfield University (RMCS Shrivenham), September 2004.

[42] Th. von Kármán and W. R. Sears. Airfoil Theory for Non-Uniform Motion. *Journal of the Aeronautical Sciences*, 5(10):379–390, August 1938.

[43] James E. McCune, Chun-Ming G. Lam, and Matthew T. Scott. Nonlinear Aerodynamics of Two-Dimensional Airfoils in Severe Maneuver. *AIAA Journal*, 28(3):385–393, March 1990.

[44] J. E. McCune and T. S. Tavares. Perspective: Unsteady wing theory – the Kármán/Sears legacy. *Transactions of the ASME: Journal of Fluids Engineering*, 115(4):548–560, 1993.

[45] S. A. Ansari, K. Knowles, and R. Żbikowski. Aerodynamic Modelling of Some Planforms for Insect-like Flapping Wings. In *The Royal Aeronautical Society CEAS Aerodynamics and Aerospace Conference*, pages 38.1–38.14, London, 10–12 June 2003. 10–12 June.

[46] William Thomson. On Vortex Motion. In *Mathematical and Physical Papers: Volume IV Hydrodynamics and General Dynamics*. Cambridge University Press, 1910. (a.k.a. Lord Kelvin).

[47] J. C. Wu. Theory for Aerodynamic Force and Moment in Viscous Flow. *AIAA Journal*, 19(4):434–441, Apr 1981.

[48] T. Maxworthy. Experiments on the Weis-Fogh Mechanism of Lift Generation by Insects in Hovering Flight. Part 1: Dynamics of the 'fling'. *Journal of Fluid Mechanics*, 93:47–63, 1979.

[49] Karl E. Gustafson and James A. Sethian, editors. *Vortex Methods and Vortex Motion*. SIAM, Philadelphia, 1991.

[50] M. H. Dickinson and K. G. Götz. Unsteady Aerodynamic Performance of Model Wings at Low Reynolds Numbers. *Journal of Experimental Biology*, 174:45–64, 1993.

[51] Coen van den Berg and Charles P. Ellington. The Vortex Wake of a "Hovering" Model Hawkmoth. *Philosophical Transactions of the Royal Society of London Series B*, 352(1351):317–328, 29 March 1997.

[52] M. H. Dickinson. 2003 California Institute of Technology, Pasadena, CA, priv. comm.

[53] J. Szmelter and R. Żbikowski. A study of flow arising from insect wing flapping motion. *International Journal for Numerical Methods in Fluids*, 40:497–505, 2002.

[54] Fumiya Togashi, Yasushi Ito, Mitsuhiro Murayama, Kazuhiro Nakahashi, and Takuma Kato. Flow Simulation of Flapping Wings of an Insect Using Overset Unstructured Grid. In *15th AIAA Computational Fluid Dynamics Conference*, volume AIAA-2001-2619, pages 1–10. AIAA, June 2001. June.